CHAPTER 5

Systems of Equations and Inequalities

Section 5A

Systems of Linear Equations

- **Technology Lab** Solve Linear Equations by Using a Spreadsheet
- **5-1 Solving Systems by Graphing**
- **Algebra Lab** Model Systems of Linear Equations
- **5-2 Solving Systems by Substitution**
- **5-3 Solving Systems by Elimination**
- **Connecting Algebra to Number Theory** Solving Classic Problems
- **5-4 Solving Special Systems**

Section 5B

Linear Inequalities

- **5-5 Solving Linear Inequalities**
- **5-6 Solving Systems of Linear Inequalities**
- **Technology Lab** Solve Systems of Linear Inequalities

Pacing Guide for 45-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 5

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
Technology Lab 5-1 Lesson	5-1 Lesson Algebra Lab	5-2 Lesson	5-3 Lesson	Connecting Algebra to Number Theory
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
5-4 Lesson	5-4 Lesson	Multi-Step Test Prep Ready to Go On?	5-5 Lesson	5-6 Lesson
DAY 11	**DAY 12**	**DAY 13**	**DAY 14**	**DAY 15**
5-6 Lesson	Technology Lab	Multi-Step Test Prep Ready to Go On?	Chapter Review	Chapter Test

Pacing Guide for 90-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 5

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
Technology Lab 5-1 Lesson Algebra Lab	5-2 Lesson 5-3 Lesson	Connecting Algebra to Number Theory 5-4 Lesson	5-4 Lesson Multi-Step Test Prep Ready to Go On?	5-5 Lesson 5-6 Lesson
DAY 6	**DAY 7**	**DAY 8**		
5-6 Lesson Technology Lab	Multi-Step Test Prep Ready to Go On? Chapter Review	Chapter Test		

ONGOING ASSESSMENT and INTERVENTION

	DIAGNOSE	PRESCRIBE
Assess Prior Knowledge	**Before the Chapter**	
	Diagnose readiness for the chapter.	Prescribe intervention.
	Are You Ready? SE	***Are You Ready? Intervention***
Formative Assessment	**Before Every Lesson**	
	Diagnose readiness for the lesson.	Prescribe intervention.
	Warm Up TE	**Reteach** CRB
	During Every Lesson	
	Diagnose understanding of lesson concepts.	Prescribe intervention.
	Check It Out! SE	**Reading Strategies** CRB
	Questioning Strategies TE	***Success for Every Learner***
	Think and Discuss SE	***Lesson Tutorial Videos***
	Write About It SE	
	Journal TE	
	After Every Lesson	
	Diagnose mastery of lesson concepts.	Prescribe intervention.
	Lesson Quiz TE	**Reteach** CRB
	Test Prep SE	**Test Prep Doctor** TE
	ExamView™ Assessment Suite	***Homework Help* Online**
	Before Chapter Testing	
	Diagnose mastery of concepts in chapter.	Prescribe intervention.
	Ready to Go On? SE	***Ready to Go On? Intervention***
	Multi-Step Test Prep SE	**Scaffolding Questions** TE
	Section Quizzes AR	**Reteach** CRB
	ExamView™ Assessment Suite	***Lesson Tutorial Videos***
	Before High Stakes Testing	
	Diagnose mastery of benchmark concepts.	Prescribe intervention.
	College Entrance Exam Practice SE	***College Entrance Exam Practice***
	Standardized Test Prep SE	
Summative Assessment	**After the Chapter**	
	Check mastery of chapter concepts.	Prescribe intervention.
	Multiple-Choice Tests (Forms A, B, C)	**Reteach** CRB
	Free-Response Tests (Forms A, B, C)	***Lesson Tutorial Videos***
	Performance Assessment AR	
	Cumulative Test AR	
	ExamView™ Assessment Suite	

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **CRB** = *Chapter Resource Book* **AR** = *Assessment Resources* Available online

CHAPTER 5

Lesson Resources

Before the Lesson

Prepare

Teacher One Stop
- Editable lesson plans
- Calendar Planner
- Easy access to all chapter resources

Lesson Transparencies
- Teacher Tools

Teach the Lesson

Introduce

Alternate Openers: Explorations

Lesson Transparencies
- Warm Up
- Problem of the Day

Teach

Lesson Transparencies
- Teaching Transparencies

Know-It Notebook™
- Vocabulary
- Key Concepts

Power Presentations

Lesson Tutorial Videos SPANISH

Interactive Online Edition
- Lesson Activities
- Lesson Tutorial Videos

Lab Activities

Lab Resources Online

Online Interactivities SPANISH

TechKeys

Practice the Lesson

Practice

Chapter Resources
- Practice A, B, C

Practice and Problem Solving Workbook

IDEA Works!® Modified Worksheets and Tests

ExamView™ *Assessment Suite*

Homework Help Online

Online Interactivities SPANISH

Interactive Online Edition
- Homework Help

Apply

Chapter Resources
- Problem Solving

Practice and Problem Solving Workbook

Interactive Online Edition
- Chapter Project

Project Teacher Support

After the Lesson

Reteach

Chapter Resources
- Reteach
- Reading Strategies ELL

Success for Every Learner

Review

Interactive Answers and Solutions

Solutions Key

Know-It Notebook™
- Big Ideas
- Chapter Review

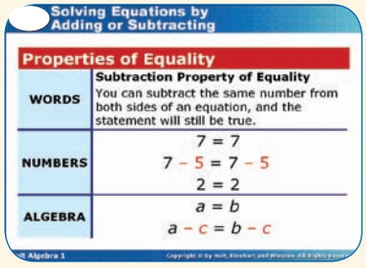
Extend

Chapter Resources
- Challenge

Technology Highlights for the Teacher

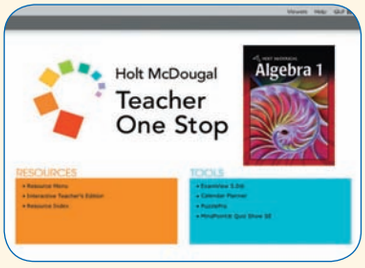
Power Presentations

Dynamic presentations to engage students. Complete PowerPoint® presentations for every lesson in this chapter.



Teacher One Stop SPANISH

Easy access to the Chapter resources and assessments. Includes lesson planning, software.



Interactive Online Edition

This chapter includes Tutorial Videos, Lesson Activities, Lesson Quizzes, Homework Help, and Chapter Project.

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* ELL English Language Learners SPANISH Spanish version available Available online

CHAPTER 5

Reaching All Learners

Teaching tips to help all learners appear throughout the chapter. A few that target specific students are included in the lists below.

All Learners

Lab Activities
Practice and Problem Solving Workbook
Know-It Notebook

Special Needs Students

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Inclusion ..... TE
IDEA Works!® Modified Worksheets and Tests
Ready to Go On? Intervention
Know-It Notebook
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

Developing Learners

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

On-Level Learners

Practice B ..... CRB
Problem Solving ..... CRB
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH

Advanced Learners

Practice C ..... CRB
Challenge ..... CRB
Challenge Exercises ..... SE
Reading and Writing Math *Extend* ..... TE
Are You Ready? Enrichment
Ready To Go On? Enrichment

English Language Learners

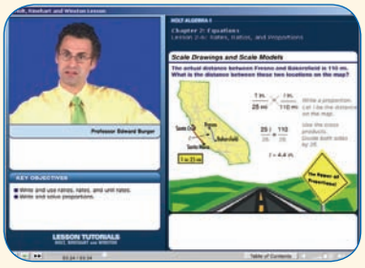
ENGLISH LANGUAGE LEARNERS

Reading Strategies ..... CRB
Are You Ready? Vocabulary ..... SE
Vocabulary Connections ..... SE
Vocabulary Review ..... SE
English Language Learners ..... TE
Success for Every Learner
Know-It Notebook
Multilingual Glossary
Lesson Tutorial Videos SPANISH

Technology Highlights for Reaching All Learners

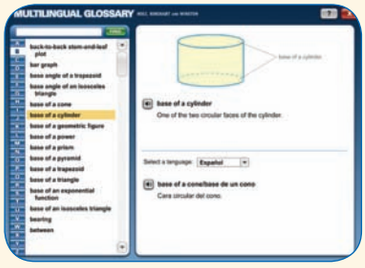
Lesson Tutorial Videos SPANISH

Starring Holt authors Ed Burger and Freddie Renfro! Live tutorials to support every lesson in this chapter.



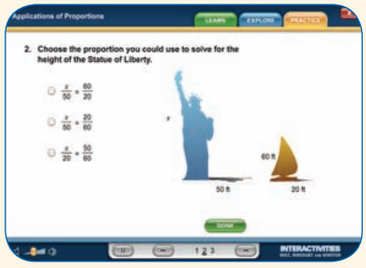
Multilingual Glossary

Searchable glossary includes definitions in English, Spanish, Vietnamese, Chinese, Hmong, Korean, and other languages.



Online Interactivities

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.



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CHAPTER 5

Ongoing Assessment

Assessing Prior Knowledge

Determine whether students have the prerequisite concepts and skills for success in the chapter.

Are You Ready? ..... SE
Warm Up ..... TE

Test Preparation

Provide review and practice for the chapter and standardized tests.

Multi-Step Test Prep ..... SE
Study Guide: Review ..... SE
Test Tackler ..... SE
Standardized Test Prep ..... SE
College Entrance Exam Practice ..... SE
Countdown to Mastery ..... SE
IDEA Works!® Modified Worksheets and Tests

Alternative Assessment

Assess students' understanding of the chapter concepts and combined problem-solving skills.

Chapter Project ..... SE
Alternative Assessment ..... TE
Performance Assessment ..... AR
Portfolio Assessment ..... AR

Lesson Assessment

Provide formative assessment for each lesson of the chapter.

Questioning Strategies ..... TE
Think and Discuss ..... SE
Check It Out! Exercises ..... SE
Write About It ..... SE
Journal ..... TE
Lesson Quiz ..... TE
Alternative Assessment ..... TE
IDEA Works!® Modified Worksheets and Tests

Weekly Assessment

Provide formative assessment for each section of the chapter.

Multi-Step Test Prep ..... SE
Ready to Go On? ..... SE
Section Quizzes ..... AR
ExamView™ Assessment Suite

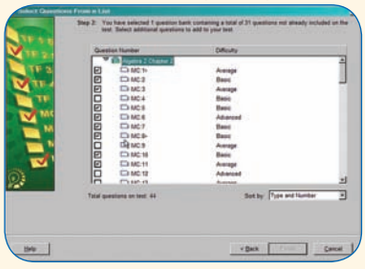
Chapter Assessment

Provide summative assessment of the chapter mastery.

Chapter Test ..... SE
Chapter Test (Levels A, B, C) ..... AR
• Multiple Choice • Free Response
Cumulative Test ..... AR
Test and Practice Generator ..... *Teacher One Stop*
IDEA Works!® Modified Worksheets and Tests

Technology Highlights for Assessment

ExamView™ Assessment Suite
Use the chapter problem banks to create assessments and worksheets to print out or deliver online. Includes dynamic problems.



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CHAPTER 5

Formal Assessment

Three levels (A, B, C) of multiple-choice and free-response chapter tests, along with a performance assessment, are available in the *Assessment Resources.*

A Chapter Test

C Chapter Test

MULTIPLE CHOICE

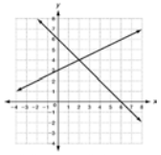
B Chapter Test

Select the best answer.

1. Which ordered pair is a solution of $\begin{cases} x - y = -3 \\ 2x + y = 0 \end{cases}$?

   A (–3, 0)
   B (–1, 2)
   C (0, 0)
   D (1, 4)

2. The graph of a system of linear equations is shown below. What is the solution of the system?



   F (0, 3) H (2, 4)
   G (0, 6) J (6, 0)

3. Solve by substitution: $\begin{cases} y = x + 3 \\ 2x + y = 9 \end{cases}$.

   A (1.5, 4.5)
   B (2, 5)
   C (3, 6)
   D (6, 9)

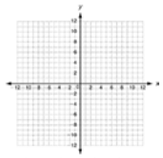
4. Solve by elimination: $\begin{cases} 3x + 2y = -1 \\ x - 2y = 11 \end{cases}$.

   F (–5, 8) H (3, –4)
   G $\left(-\frac{5}{2}, \frac{17}{4}\right)$ J $\left(4, -\frac{7}{2}\right)$

5. Sam needs to make a long-distance call from a pay phone. With his prepaid phone card, he will be charged $1.00 to connect and $0.50 per minute. If he places a collect call with the operator he will be charged $3.00 to connect and $0.25 per minute. In how many minutes will the phone card and the collect call cost the same?

   A 5 min
   B $5\frac{1}{5}$ min
   C 8 min
   D 16 min

6. Solve by any method: $\begin{cases} 2x + y = 11 \\ x + 3y = 18 \end{cases}$.



   F $\left(-\frac{15}{7}, \frac{47}{7}\right)$ H $\left(\frac{7}{2}, \frac{15}{4}\right)$
   G (3, 5) J (5, 1)

7. Which term describes a system with no solution?

   A consistent
   B dependent
   C inconsistent
   D independent

B Chapter Test *(continued)*

8. Which equation would make this system consistent and dependent?

   $\begin{cases} y = 2x - 4 \\ _____ \end{cases}$

   F $y = 2x + 5$ H $2x + y = -4$
   G $6x - 3y = 12$ J $y = 3x - 4$

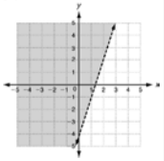
9. Solve $\begin{cases} 4x = 2y + 6 \\ y = 2x - 3 \end{cases}$.

   A no solution
   B (0, 0)
   C $\left(\frac{3}{2}, 0\right)$
   D infinitely many solutions

10. Which ordered pair is a solution of $y > 5x - 2$?

    F (1, 5) H (3, 13)
    G (2, 7) J (4, 4)

11. The solution of which linear inequality is graphed below?

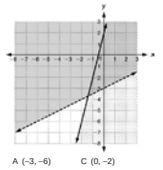


    A $3x - y > -4$ C $-3x + y \geq -4$
    B $3x - y < 4$ D $3x + y \leq 4$

12. For which system of linear inequalities is the ordered pair (–1, 3) a solution?

    F $\begin{cases} y \leq -x + 6 \\ y < 2x - 3 \end{cases}$ H $\begin{cases} y \geq -x + 6 \\ y < 2x + 3 \end{cases}$
    G $\begin{cases} y > -x + 6 \\ y \geq 2x - 3 \end{cases}$ J $\begin{cases} y \leq -x + 6 \\ y > 2x - 3 \end{cases}$

13. The graph of a system of linear inequalities is shown below. Which ordered pair is a solution of the system?



    A (–3, –6) C (0, –2)
    B (–2, 0) D (3, –6)

14. A factory makes widgets x and gadgets y. Widgets cost $3 and gadgets cost $5 each to make. The boss wants at least 10 widgets and 20 gadgets per day. Total costs need to be at most $300 per day. What system represents this situation?

    F $\begin{cases} x \geq 10 \\ y \geq 20 \\ x + y \leq 300 \end{cases}$ H $\begin{cases} x \geq 10 \\ y \geq 20 \\ 3x + 5y \leq 300 \end{cases}$
    G $\begin{cases} x \geq 10 \\ y \geq 20 \\ 3x + 5y \geq 300 \end{cases}$ J $\begin{cases} x \geq 10 \\ y \geq 20 \\ 5x + 3y \leq 300 \end{cases}$

A Chapter Test

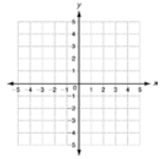
C Chapter Test

FREE RESPONSE

B Chapter Test

1. Tell whether (–5, –6) is a solution of $\begin{cases} x - 2y = 7 \\ y - x = -1 \end{cases}$.

2. Solve $\begin{cases} 4x + y = -1 \\ y - 4 = x \end{cases}$ by graphing.

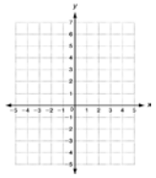


3. Solve by substitution: $\begin{cases} 3x - 2 = y \\ y - 2x = -5 \end{cases}$.

4. Solve by elimination: $\begin{cases} 3x - 2y = 4 \\ x + 4y = 34 \end{cases}$.

5. The population of Medford High School is 800 students and the population of Westville High School is 1240 students. The population of Medford High has been increasing at a rate of 30 students per year, while the population of Westville High has been decreasing by 25 students per year. If these rates remain constant, in how many years will the populations of the two schools be the same? How many students will that be?

6. Solve by any method: $\begin{cases} y - 5 = -\frac{1}{2}x \\ x = y - 2 \end{cases}$.



7. Classify $\begin{cases} y = \frac{2}{3}x + 7 \\ 3y = 2x - 3 \end{cases}$.

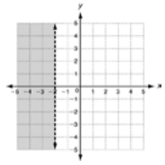
B Chapter Test *(continued)*

8. How many solutions does $\begin{cases} -4x + y = 1 \\ 2y - 2 = 8x \end{cases}$ have?

9. Solve $\begin{cases} 8y - 6x = 48 \\ 2y = \frac{3}{2}x - 12 \end{cases}$.

10. Tell whether (4, 3) is a solution of $y \leq -2x - 5$.

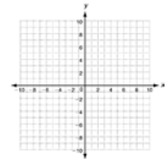
11. Write the inequality represented by the graph below.



12. Tell whether (–1, –4) is a solution of $\begin{cases} -2x - y > 6 \\ y \leq 4x \end{cases}$.

13. Graph $\begin{cases} -4x + y > -6 \\ 5y \leq -x - 20 \end{cases}$.

    Give two ordered pairs that are solutions and two ordered pair that are not solutions.



14. A farmer has 165 feet of fencing material in which to enclose a rectangular grazing area. He wants the length x to be greater than 50 feet and the width y to be no more than 20 feet. Write a system to represent this situation.

PERFORMANCE ASSESSMENT

Chapter Test

The Marathon

Purpose

This performance task assesses the student's ability to graph a system of linear equations and to solve the system both graphically and algebraically.

Time

35–40 minutes

Grouping

Individuals or pairs

Preparation Hints

Review the definitions for *system of linear equations* and *solution of a system of linear equations*. Discuss the three methods of solving systems taught in Chapter 6: graphing, substitution, and elimination.

Overview

When two runners compete in a marathon, their distance y from the starting line as a function of time x can be represented by a linear equation in slope-intercept form: $y = (speed)x + (starting\ position)$. Solving the system of equations gives the time and distance when the runners meet.

Introduce the Task

Ask whether any students have run in a marathon, half-marathon, or foot race. If so, have them describe the length of the course, what they did to train for the race, and how many people competed.

Explain that in large marathons, it is impossible for the thousands of participants to all start at the starting line. Some marathons use staggered start times or distances, frequently based on ability.

Performance Indicators

_____ Writes linear equations in slope-intercept form.
_____ Graphs a system of linear equations.
_____ Estimates the solution of a system of linear equations from a graph.
_____ Uses substitution or elimination to find the solution of a system.
_____ Interprets the meaning of the solution of the system in context.

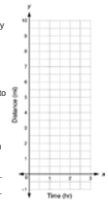
Scoring Rubric

Level 4: Student solves all problems correctly and gives good explanations.
Level 3: Student solves all problems but does not give satisfactory explanations.
Level 2: Student solves some problems but does not give satisfactory explanations.
Level 1: Student is not able to solve any of the problems.

Chapter Test *(continued)*

The Marathon

Bethany and Calista are sisters who both run marathons. Today they are racing against each other in the same marathon. Because there are thousands of people racing, Bethany and Calista are assigned random starting positions. Bethany starts at the starting line, while Calista starts a half-mile behind the starting line.



Calista runs one mile in 12 minutes, while Bethany runs one mile in 15 minutes. So, although Calista starts behind Bethany, she hopes to pass her sister at some point during the race.

Let x represent the amount of time in hours that Bethany or Calista run and let y represent distance after the starting line in miles.

1. The rate or speed at which someone runs is frequently stated in miles per hour.
   a. What is Bethany's speed in miles per hour? _____
   b. What is Calista's speed in miles per hour? _____
2. Write a linear equation in slope-intercept form that describes...
   a. Bethany's distance as a function of time. _____
   b. Calista's distance as a function of time. _____
3. On the grid above, graph the system of equations that you wrote for question 2.
4. Use your graph to estimate...
   a. who is in the lead after 15 minutes (0.25 hour). _____
   b. the time when Calista will catch up to Bethany. _____
   c. how far after the starting line the sisters catch up to each other. _____
   d. who is in the lead after 2 hours if each sister keeps running at a steady pace. _____
5. Use substitution or elimination to solve the system of equations that you wrote for question 2. Interpret the solution in terms of this problem. _____
6. A marathon is 26.2 miles. Which sister do you think will cross the finish line first? Explain. _____

CHAPTER 5

MATHEMATICAL PRACTICES **Reason abstractly and quantitatively.**

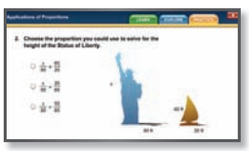
SECTION 5A
Systems of Linear Equations

MULTI-STEP TEST PREP Students write and solve systems of equations to help them make good business decisions.

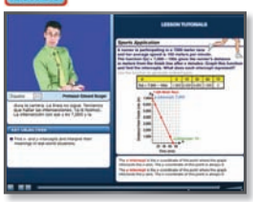
SECTION 5B
Linear Inequalities

MULTI-STEP TEST PREP Students write and graph systems of inequalities to model a real-world sales situation.

Interactivities Online ▶



Video Lesson Tutorials Online



Lesson Tutorial Videos are available for EVERY example.

CHAPTER 5

Systems of Equations and Inequalities

COMMON CORE

5A Systems of Linear Equations

Lab	Solve Linear Equations by Using a Spreadsheet	A.REI.3
5-1	Solving Systems by Graphing	A.REI.6
Lab	Model Systems of Linear Equations	A.REI.6
5-2	Solving Systems by Substitution	A.REI.6
5-3	Solving Systems by Elimination	A.REI.5
5-4	Solving Special Systems	A.REI.6

5B Linear Inequalities

5-5	Solving Linear Inequalities	A.REI.12
5-6	Solving Systems of Linear Inequalities	A.REI.12
Lab	Solve Systems of Linear Inequalities	A.REI.12

Chapter Focus

- Solve real-world problems involving systems of linear equations and inequalities.

Where's *the Money?*

You can solve a system of equations to determine how many basketball game tickets you can buy at different price levels.

Learn It Online
Chapter Project Online

Glenn James/NBAE/Getty Images

Chapter Project

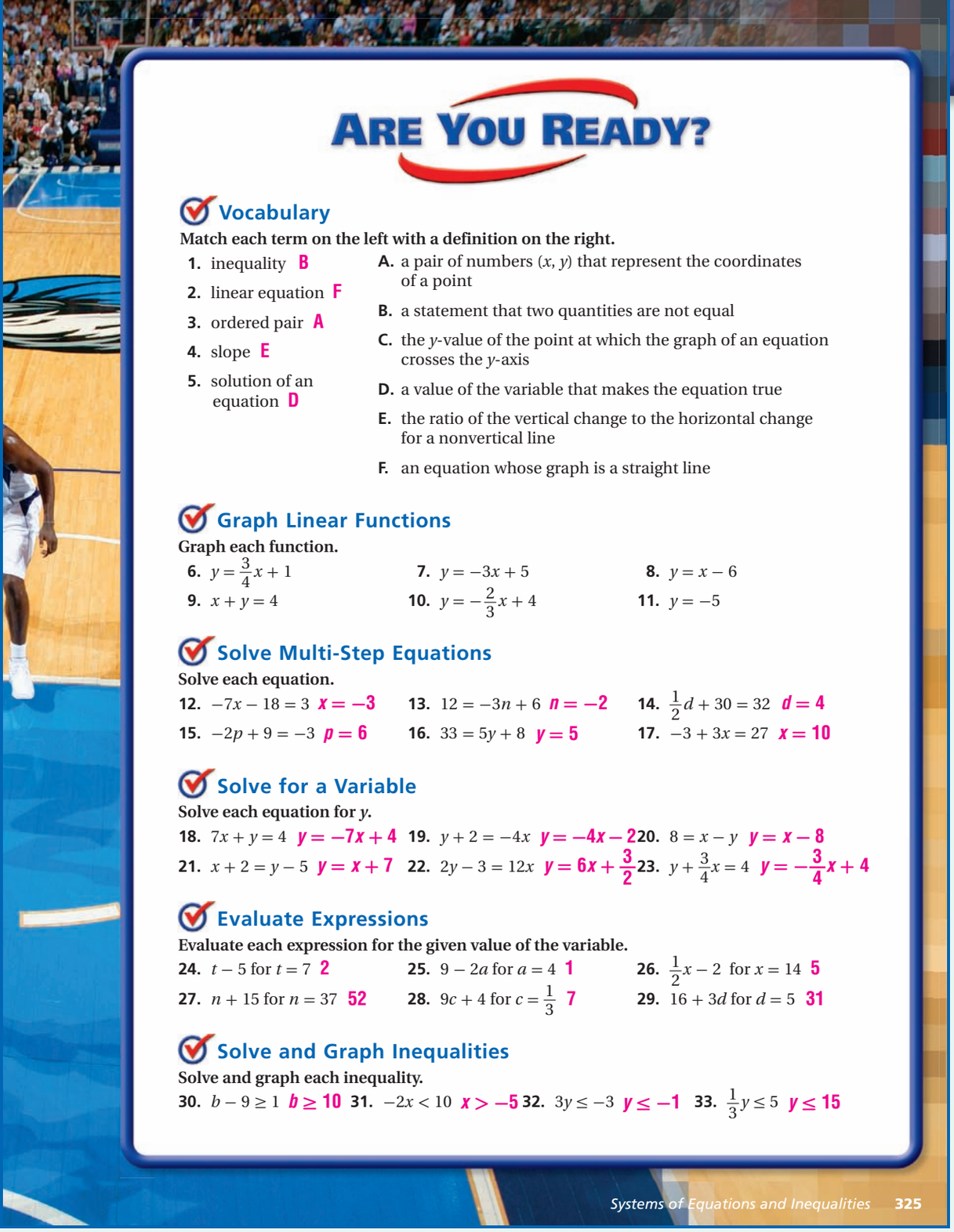
A project for this chapter is available online.

Project Resources

All project resources for teachers and students are provided online.

Learn It Online
Chapter Project Online

324 Chapter 5



ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. inequality **B**
2. linear equation **F**
3. ordered pair **A**
4. slope **E**
5. solution of an equation **D**

A. a pair of numbers (x, y) that represent the coordinates of a point

B. a statement that two quantities are not equal

C. the y-value of the point at which the graph of an equation crosses the y-axis

D. a value of the variable that makes the equation true

E. the ratio of the vertical change to the horizontal change for a nonvertical line

F. an equation whose graph is a straight line

Graph Linear Functions

Graph each function.

6. $y = \frac{3}{4}x + 1$
7. $y = -3x + 5$
8. $y = x - 6$
9. $x + y = 4$
10. $y = -\frac{2}{3}x + 4$
11. $y = -5$

Solve Multi-Step Equations

Solve each equation.

12. $-7x - 18 = 3$ **$x = -3$**
13. $12 = -3n + 6$ **$n = -2$**
14. $\frac{1}{2}d + 30 = 32$ **$d = 4$**
15. $-2p + 9 = -3$ **$p = 6$**
16. $33 = 5y + 8$ **$y = 5$**
17. $-3 + 3x = 27$ **$x = 10$**

Solve for a Variable

Solve each equation for y.

18. $7x + y = 4$ **$y = -7x + 4$**
19. $y + 2 = -4x$ **$y = -4x - 2$**
20. $8 = x - y$ **$y = x - 8$**
21. $x + 2 = y - 5$ **$y = x + 7$**
22. $2y - 3 = 12x$ **$y = 6x + \frac{3}{2}$**
23. $y + \frac{3}{4}x = 4$ **$y = -\frac{3}{4}x + 4$**

Evaluate Expressions

Evaluate each expression for the given value of the variable.

24. $t - 5$ for $t = 7$ **2**
25. $9 - 2a$ for $a = 4$ **1**
26. $\frac{1}{2}x - 2$ for $x = 14$ **5**
27. $n + 15$ for $n = 37$ **52**
28. $9c + 4$ for $c = \frac{1}{3}$ **7**
29. $16 + 3d$ for $d = 5$ **31**

Solve and Graph Inequalities

Solve and graph each inequality.

30. $b - 9 \geq 1$ **$b \geq 10$**
31. $-2x < 10$ **$x > -5$**
32. $3y \leq -3$ **$y \leq -1$**
33. $\frac{1}{3}y \leq 5$ **$y \leq 15$**

CHAPTER 5

ARE YOU READY?

Organizer

Objective: Assess students' understanding of prerequisite skills.

Assessing Prior Knowledge

INTERVENTION

Diagnose and Prescribe

Use this page to determine whether intervention is necessary or whether enrichment is appropriate.

Resources

- ***Are You Ready? Intervention and Enrichment* Worksheets**
- ***Are You Ready?* Online**

Answers

6–11. See Additional Answers.

30–33. For graphs, see Additional Answers.

ARE YOU READY?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Prerequisite Skill	*Are You Ready? Intervention,* Chapter 5 Worksheets	*Are You Ready? Intervention,* Chapter 5 Online	
Graph Linear Functions	Skill 75	Activity 75	Diagnose and Prescribe Online
Solve Multi-Step Equations	Skill 69	Activity 69	
Solve for a Variable	Skill 72	Activity 72	
Evaluate Expressions	Skill 60	Activity 60	
Solve and Graph Inequalities	Skill 74	Activity 74	

***Are You Ready? Enrichment,* Chapter 5**
Worksheets
Online

Organizer

Objective: Help students organize the new concepts they will learn in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Possible answers to *Vocabulary Connections*

1. Possible answer: A system of linear equations is a group of linear equations, rather than just one linear equation.
2. none
3. independent system
4. any ordered pair that makes the inequality true

CHAPTER 5 Study Guide: Preview

Where You've Been

Previously, you

- solved one-step and multi-step equations.
- solved one-step and multi-step inequalities.
- graphed linear equations on a coordinate plane.

In This Chapter

You will study

- how to find a solution that satisfies two linear equations.
- how to find solutions that satisfy two linear inequalities.
- how to graph one or more linear inequalities on a coordinate plane.

Where You're Going

You can use the skills in this chapter

- to determine which purchases are better deals.
- in other classes, such as Economics and Chemistry.
- to solve linear equations that involve three or more variables in future math classes.

Key Vocabulary/Vocabulario

consistent system	sistema consistente
dependent system	sistema dependiente
inconsistent system	sistema inconsistente
independent system	sistema independiente
linear inequality	desigualdad lineal
solution of a linear inequality	solución de una desigualdad lineal
system of linear equations	sistema de ecuaciones lineales

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *system* means "a group." How do you think a **system of linear equations** is different from a linear equation?
2. A **consistent system** has *at least one* solution. How many solutions do you think an **inconsistent system** has?
3. A **dependent system** has infinitely many solutions. Which vocabulary term above means a system with *exactly one* solution?
4. A solution of a linear equation is an ordered pair that makes the equation true. Modify this to define **solution of a linear inequality**.

MATHEMATICAL PRACTICES

The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. **Make sense of problems and persevere in solving them.** Pages 331, 332–334, 340–342, 347–349, 355–356, 358, 364–365, 370–371, 374
2. **Reason abstractly and quantitatively.** Pages 346, 356, 369
3. **Construct viable arguments and critique the reasoning of others.** Pages 331, 333, 339, 341, 347, 348–349, 355, 356–357, 363, 365–366, 369, 371–372
4. **Model with mathematics.** Pages 331, 339, 354, 362
5. **Use appropriate tools strategically.** Pages 328, 333, 335, 336, 343, 346, 361, 373
6. **Attend to precision.** Pages 342, 372
7. **Look for and make use of structure.** Pages 348, 357, 372, 353
8. **Look for and express regularity in repeated reasoning.** Pages 355–357

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Writing Strategy: Write a Convincing Argument/Explanation

The Write About It icon appears throughout the book. These icons identify questions that require you to write a complete argument or explanation. Writing a convincing argument or explanation shows that you have a solid understanding of a concept.

To be effective, an argument or explanation should include

- evidence, work, or facts.
- a complete response that will answer or explain.

23. **Write About It** Lewis invested $1000 at 3% simple interest for 4 years. Lisa invested $1000 at 4% simple interest for 3 years. Explain why Lewis and Lisa earned the same amount of interest.

Step 1 **Identify what you need to answer or explain.**
Explain why Lewis and Lisa earned the same amount of interest.

Step 2 **Give evidence, work, or facts that are needed to answer the question.**
Use the formula for simple interest to find the amount of interest earned: $I = Prt$.

Lewis: $P = 1000$, $r = 0.03$, $t = 4$

$I = Prt = 1000(0.03)(4) = 120$

$I = 1000(0.12) = \$120$

Lisa: $P = 1000$, $r = 0.04$, $t = 3$

$I = Prt = 1000(0.04)(3) = 120$

$I = 1000(0.12) = \$120$

Step 3 **Write a complete response that answers or explains.**
Lewis and Lisa both invested the same amount of money, $1000. They earned the same amount of interest because 0.04×3 and 0.03×4 both equal 0.12. They both earned $0.12 \times \$1000$, or $120.

Try This

Write a convincing argument or explanation.

1. What is the least whole number that is a solution of $12x + 15.4 > 118.92$? Explain.
2. Which equation has an error? Explain the error.

   A. $4(6 \cdot 5) = (4)6 \cdot (4)5$ **B.** $4(6 \cdot 5) = (4 \cdot 6)5$

COMMON CORE **Standards for Mathematical Content** High School

		LESSON
CREATING EQUATIONS Solve equations and inequalities in one variable		
CC.9-12.A.REI.3	Create equations in two ore more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	TLab 5-1
CREATING EQUATIONS Solve systems of equations		
CC.9-12.A.REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	5-3
CC.9-12.A.REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	5-1, Lab 5-2, 5-2, 5-3, 5-4
CREATING EQUATIONS Represent and solve equations and inequalities graphically		
CC.9-12.A.REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	5-5, 5-6, TLab 5-6

Organizer

Objective: Help students apply strategies to understand and retain key concepts.

INTERACTIVE **Online Edition**

Writing Strategy: Write a Convincing Argument/Explanation

Discuss A convincing argument or explanation leaves no doubt in the mind of the reader. There is no point at which the reader can say "Why?" or "How did you get that?"

Extend As students work through the *Write About It* problems in this chapter, encourage them to write as thoroughly as possible. Have students check each other's work, checking for adequate evidence and completed work shown.

Answers to *Try This*

Possible answers:

1. The actual answer is $x > 8.62\overline{6}$. The nearest whole number that is greater than $8.62\overline{6}$ is 9.
2. Equation A is the equation with the error. The Distributive Property does not work over multiplication, only over addition or subtraction.

Reading Connection

Mathematicians Are People, Too: Stories from the Lives of Great Mathematicians, Volume 2 **by Luetta & Wilbert Reimer**

Here are intriguing tales of 15 ground-breaking mathematicians. The authors stress the human side of these men and women, almost (but not completely!) convincing us that they're just like us.

Activity After students have solved systems by graphing, have them read Chapter 5, *René Descartes.* Ask them to report on Descartes's development of the Cartesian coordinate system, and to judge whether the story that the idea occurred to Descartes while watching a fly on the ceiling is myth or reality.

SECTION 5A

Systems of Linear Equations

One-Minute Section Planner

Lesson	Lab Resources	Materials
Technology Lab Solve Linear Equations by Using a Spreadsheet • Use a spreadsheet to model real-world linear relationships. ☐ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** spreadsheet software **Optional** graphing calculator
Lesson 5-1 Solving Systems by Graphing • Identify solutions of systems of linear equations in two variables. • Solve systems of linear equations in two variables by graphing. ☐ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Required** graph paper **Optional** graphing calculator
Algebra Lab Model Systems of Linear Equations • Use algebra tiles to model and solve systems of linear equations. ☐ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Required** algebra tiles (MK)
Lesson 5-2 Solving Systems by Substitution • Solve systems of linear equations in two variables by substitution. ☐ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☐ SAT Subject Tests		**Optional** graphing calculator
Lesson 5-3 Solving Systems by Elimination • Solve systems of linear equations in two variables by elimination. • Compare and choose an appropriate method for solving systems of linear equations. ☐ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** graphing calculator, transparent algebra tiles (MK), algebra tiles (MK)
Lesson 5-4 Solving Special Systems • Solve special systems of linear equations in two variables. • Classify systems of linear equations and determine the number of solutions. ☐ SAT-10 ☐ NAEP ☑ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** graphing calculator

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 1: 1, 3, 5, 7, 9, 11, 13, 15, 17a, 17b, 17c, 19, 21, 23, 25, 29, 31

Lesson 2: 1, 3, 5, 7a, 7b, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 35, 37, 39, 41

Lesson 3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 25, 27 29, 31a, 31b, 31c, 33, 35a, 35b, 37, 39

Lesson 4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 27, 29, 31, 33, 35

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

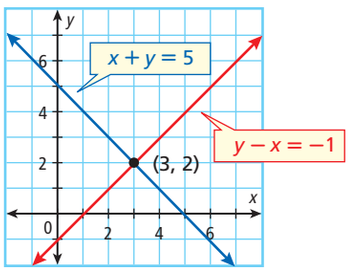
COMMON CORE PROFESSIONAL DEVELOPMENT

SYSTEMS OF LINEAR EQUATIONS

Lessons 5-1 to 5-4

A *system of linear equations* is a set of two or more linear equations that each contain two or more variables. In this course, the systems consist of two equations that each contain two variables. A *solution* of such a system is an ordered pair that satisfies both equations. Consider the system $\begin{cases} x + y = 5 \\ y - x = -1 \end{cases}$. The ordered pair (3, 2) is a solution of the system because it satisfies both equations: $3 + 2 = 5$ and $2 - 3 = -1$.

The graph of an equation is the set of all of its solutions. Therefore, if an ordered pair is a solution of two linear equations, it must lie on both graphs. In other words, the solution is the *intersection* of the graphs. The intersection of the graphs of $x + y = 5$ and $y - x = -1$ is (3, 2).



This graphic representation leads naturally to the conclusions that some systems may have no solutions (lines may be parallel) and that some systems may have infinitely many solutions (lines may be coincident).

Graphing is an intuitive way to solve systems, and it visually reinforces the meaning of a solution. Graphing also highlights the connection between algebra and geometry; in particular, it shows that you can solve a problem presented in purely algebraic terms by using geometric methods.

ALGEBRAIC METHODS

Lessons 5-2 to 5-4

The intersection point of two lines may not have integer coordinates, thereby motivating the need for algebraic methods of solving systems. In general, the goal of algebraic methods is to eliminate a variable so that the techniques for solving single-variable equations may be used.

As an example of an algebraic method, one can solve both equations in a system for y to get two equations that can be set equal to each other. Solving both equations for y in the system discussed earlier gives $y = 5 - x$ and $y = x - 1$. The two expressions for y must be equal, so $5 - x = x - 1$. Solving shows that $x = 3$, and substituting this value of x into either of the original equations gives $y = 2$.

This method is called the *substitution method*. In general, solve either equation for either variable and then substitute into the other equation. This yields an equation that can be solved using previously learned skills. In future math courses, students will see that the substitution method is useful for solving systems when one or more of the equations are not linear.

Although the substitution method works with any system, it may be awkward to use when no variable term has a coefficient of 1. In this case, the *elimination method* may be a better choice.

Students should understand that the elimination method is based on the properties of equality. For example, to solve $\begin{cases} 2x + 3y = 11 \\ -6x + 5y = -19 \end{cases}$, you can multiply both sides of the first equation by 3 using the Multiplication Property of Equality. To solve the resulting system, $\begin{cases} 6x + 9y = 33 \\ -6x + 5y = -19 \end{cases}$, you can add the two equations together to eliminate the x-terms. This is justified by the Addition Property of Equality since you are adding equal quantities, $-6x + 5y$ and -19, to both sides of the first equation. The result is one equation in one variable, $14y = 14$, that is easy to solve for y. Then substitute to find x.

In general, elimination works well for systems of all linear equations containing any number of variables, and it is a fundamental process in the study of linear algebra.

Technology LAB Organizer

Use with the lesson *Solving Systems by Graphing*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a spreadsheet to model real-world linear relationships.

Materials: spreadsheet software

Countdown

Lab Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

Ask students what the breakeven point represents. Lead them to see that it is when the cost and sales are equal and profit is thus 0.

Close

Key Concept

You can use a spreadsheet to view the expressions evaluated for many values at a time and observe the pattern.

Assessment

Journal Have students explain what a breakeven point is and how, given the cost and sales equations, they can use a spreadsheet to find this point.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Learn It Online
State Resources Online

5-1 Technology LAB

Solve Linear Equations by Using a Spreadsheet

You can use a spreadsheet to answer "What if…?" questions. By changing one or more values, you can quickly model different scenarios.

Use with Solving Systems by Graphing

Learn It Online
Lab Resources Online

Activity

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Company Z makes DVD players. The company's costs are $400 per week plus $20 per DVD player. Each DVD player sells for $45. How many DVD players must company Z sell in one week to make a profit?

Let n represent the number of DVD players company Z sells in one week.

$c = 400 + 20n$ *The total cost is $400 plus $20 times the number of DVD players made.*

$s = 45n$ *The total sales income is $45 times the number of DVD players sold.*

$p = s - c$ *The total profit is the sales income minus the total cost.*

1. Set up your spreadsheet with columns for number of DVD players, total cost, total income, and profit.
2. Under Number of DVD Players, enter 1 in cell A2.
3. Use the equations above to enter the formulas for total cost, total sales, and total profit in row 2.
   - In cell B2, enter the formula for total cost.
   - In cell C2, enter the formula for total sales income.
   - In cell D2, enter the formula for total profit.
4. Fill columns A, B, C, and D by selecting cells A1 through D1, clicking the small box at the bottom right corner of cell D2, and dragging the box down through several rows.
5. Find the point where the profit is $0. This is known as the breakeven point, where total cost and total income are the same.

Company Z Profit

	A	B	C	D
1	Number of DVD Players	Total Cost ($)	Total Income ($)	Profit ($)
2	1	420	45	-375

= 400 + 20*A2 = 45*A2 = C2 – B2

Company Z Profit

	A	B	C	D
1	Number of DVD Players	Total Cost ($)	Total Income ($)	Profit ($)
16	15	700	675	-25
17	16	720	720	0
18	17	740	765	25

Breakeven point

Profit begins.

Company Z must sell 17 DVD players to make a profit. The profit is $25.

Try This

For Exercises 1 and 2, use the spreadsheet from the activity.

1. If company Z sells 10 DVD players, will they make a profit? Explain. What if they sell 16?
2. Company Z makes a profit of $225 dollars. How many DVD players did they sell? **25**

For Exercise 3, make a spreadsheet.

3. Company Y's costs are $400 per week plus $20 per DVD player. They want the breakeven point to occur with sales of 8 DVD players. What should the sales price be? **$70**

Answers to *Try This*

1. Company Z will not make a profit; it will lose $150. If it sells 16 DVD players it still will not make a profit; it will only break even.

COMMON CORE

5-1 Solving Systems by Graphing

CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. *Also* **CC.9-12.A.REI.11***, **CC.9-12.A.CED.2***, **CC.9-12.A.CED.3***

Objectives

Identify solutions of systems of linear equations in two variables.

Solve systems of linear equations in two variables by graphing.

Vocabulary

system of linear equations

solution of a system of linear equations

Why learn this?

You can compare costs by graphing a system of linear equations. (See Example 3.)

Sometimes there are different charges for the same service or product at different places. For example, Bowl-o-Rama charges $2.50 per game plus $2 for shoe rental while Bowling Pinz charges $2 per game plus $4 for shoe rental. A *system of linear equations* can be used to compare these charges.

A **system of linear equations** is a set of two or more linear equations containing two or more variables. A **solution of a system of linear equations** with two variables is an ordered pair that satisfies each equation in the system. So, if an ordered pair is a solution, it will make both equations true.

EXAMPLE 1 Identifying Solutions of Systems

Tell whether the ordered pair is a solution of the given system.

A $(4, 1); \begin{cases} x + 2y = 6 \\ x - y = 3 \end{cases}$

$x + 2y = 6$	
$4 + 2(1)$	6
$4 + 2$	6
6	6 ✓

$x - y = 3$	
$4 - 1$	3
3	3 ✓

Substitute 4 for x and 1 for y in each equation in the system.

The ordered pair $(4, 1)$ makes both equations true.

$(4, 1)$ is a solution of the system.

Helpful Hint

If an ordered pair does not satisfy the first equation in the system, there is no need to check the other equations.

B $(-1, 2); \begin{cases} 2x + 5y = 8 \\ 3x - 2y = 5 \end{cases}$

$2x + 5y = 8$	
$2(-1) + 5(2)$	8
$-2 + 10$	8
8	8 ✓

$3x - 2y = 5$	
$3(-1) - 2(2)$	5
$-3 - 4$	5
-7	5 ✗

Substitute −1 for x and 2 for y in each equation in the system.

The ordered pair $(-1, 2)$ makes one equation true, but not the other.

$(-1, 2)$ is not a solution of the system.

CHECK IT OUT!

Tell whether the ordered pair is a solution of the given system.

1a. $(1, 3); \begin{cases} 2x + y = 5 \\ -2x + y = 1 \end{cases}$ yes

1b. $(2, -1); \begin{cases} x - 2y = 4 \\ 3x + y = 6 \end{cases}$ no

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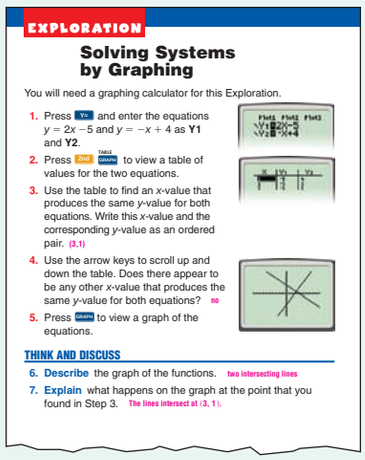
1 Introduce



EXPLORATION

Solving Systems by Graphing

You will need a graphing calculator for this Exploration.

1. Press Y= and enter the equations $y = 2x - 5$ and $y = -x + 4$ as **Y1** and **Y2**.
2. Press 2ND TABLE to view a table of values for the two equations.
3. Use the table to find an *x*-value that produces the same *y*-value for both equations. Write this *x*-value and the corresponding *y*-value as an ordered pair. (3, 1)
4. Use the arrow keys to scroll up and down the table. Does there appear to be any other *x*-value that produces the same *y*-value for both equations? no
5. Press GRAPH to view a graph of the equations.

THINK AND DISCUSS

6. **Describe** the graph of the functions. two intersecting lines
7. **Explain** what happens on the graph at the point that you found in Step 3. The lines intersect at (3, 1).

Motivate

Have students choose an ordered pair that is a solution of $y = x + 1$. Point out that there are infinitely many ordered-pair solutions. Then ask which of the ordered pairs is also a solution of $y = 5 - x$. Let them try several pairs. Tell students that there is only one ordered pair that is a solution of both equations. $(2, 3)$ Then explain that this lesson will show a method for finding that pair.

Explorations and answers are provided in your online resources.

5-1 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Identify solutions of systems of linear equations in two variables.

Solve systems of linear equations in two variables by graphing.

Online Edition
Tutorial Videos, Interactivity, Graphing Calculator

Countdown

Power Presentations with PowerPoint®

Warm Up

Evaluate each expression for $x = 1$ and $y = -3$.

1. $x - 4y$ 13
2. $-2x + y$ −5

Write each equation in slope-intercept form.

3. $y - x = 1$ $y = x + 1$
4. $2x + 3y = 6$ $y = -\frac{2}{3}x + 2$
5. $0 = 5y + 5x$ $y = -x$

Also available online

Math Humor

Teacher: What is the name of the formula that describes the phases of the moon?

Student: The lunear (linear) equation

COMMON CORE

CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

CC.9-12.A.REI.11 Explain why the *x*- coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values,….*

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

Learn It Online
State Resources Online

Power Presentations with PowerPoint®

Additional Examples

Example 1

Tell whether the ordered pair is a solution of the given system.

A. $(5, 2)$; $\begin{cases} \frac{2}{5}x - y = 0 \\ 3x - y = 13 \end{cases}$ yes

B. $(-2, 2)$; $\begin{cases} x + 3y = 4 \\ -x + y = 2 \end{cases}$ no

Example 2

Solve each system by graphing. Check your answer.

A. $\begin{cases} y = x \\ y = -2x - 3 \end{cases}$ $(-1, -1)$

B. $\begin{cases} y = x - 6 \\ y + \frac{1}{3}x = -1 \end{cases}$ $\left(\frac{15}{4}, -\frac{9}{4}\right)$

Also available online

Questioning Strategies

EXAMPLE 1

- How do you know when an ordered pair is a solution of a system of linear equations?

EXAMPLE 2

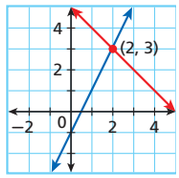
- What does the intersection of the two lines represent?

Teaching Tip **Technology** In **Example 2B,** make sure that students know how to find the point of intersection on a graphing calculator.

After both lines are graphed, press 2nd CALC TRACE and then choose **5.**

All solutions of a linear equation are on its graph. To find a solution of a system of linear equations, you need a point that each line has in common. In other words, you need their point of intersection.

$\begin{cases} y = 2x - 1 \\ y = -x + 5 \end{cases}$



The point $(2, 3)$ is where the two lines intersect and is a solution of both equations, so $(2, 3)$ is the solution of the system.

EXAMPLE 2 Solving a System of Linear Equations by Graphing

Solve each system by graphing. Check your answer.

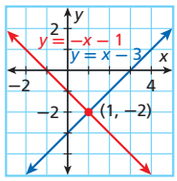
Helpful Hint

Sometimes it is difficult to tell exactly where the lines cross when you solve by graphing. It is good to confirm your answer by substituting it into both equations.

A $\begin{cases} y = x - 3 \\ y = -x - 1 \end{cases}$

Graph the system.

The solution appears to be at $(1, -2)$.



Check

Substitute $(1, -2)$ into the system.

$y = x - 3$	
-2	$1 - 3$
-2	-2 ✓

$y = -x - 1$	
-2	$-1 - 1$
-2	-2 ✓

The solution is $(1, -2)$.

B $\begin{cases} x + y = 0 \\ y = -\frac{1}{2}x + 1 \end{cases}$

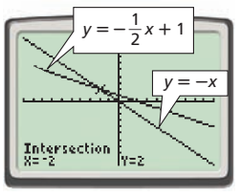
$x + y = 0$

$-x \quad -x$

$y = -x$

Rewrite the first equation in slope-intercept form.

Graph using a calculator and then use the intersection command.



Check Substitute $(-2, 2)$ into the system.

$x + y = 0$	
$-2 + 2$	0
0	0 ✓

$y = -\frac{1}{2}x + 1$	
2	$-\frac{1}{2}(-2) + 1$
2	$1 + 1$
2	2 ✓

The solution is $(-2, 2)$.

CHECK IT OUT! **Solve each system by graphing. Check your answer.**

2a. $\begin{cases} y = -2x - 1 \\ y = x + 5 \end{cases}$ $(-2, 3)$

2b. $\begin{cases} y = \frac{1}{3}x - 3 \\ 2x + y = 4 \end{cases}$ $(3, -2)$

2 Teach

Guided Instruction

Before teaching students to solve a system by graphing, review slope-intercept form and how to write an equation in that form.

Teaching Tip **Communicating Math** Warn students that the brace, {, is not always used to indicate a system of equations. A standardized test might omit the symbol and simply say that the equations are a system.

Reaching All Learners

Through Multiple Representations

Have students copy and complete the table for the system of equations.

$\begin{cases} y = -2x + 9 \\ y = x + 3 \end{cases}$

x	$y_1 = -2x + 9$	$y_2 = x + 3$
0	9	3
1	7	4
2	5	5
3	3	6

Graph the system, and discuss similarities between the table and the graph. For example, from the table, you can see that when $x = 0$, the y-value of one equation is 9, and the y-value of the other equation is 3. On the graph you can see that the point $(0, 9)$ is on one line and the point $(0, 3)$ is on the other line.

EXAMPLE 3 Problem-Solving Application

Bowl-o-Rama charges $2.50 per game plus $2 for shoe rental, and Bowling Pinz charges $2 per game plus $4 for shoe rental. For how many games will the cost to bowl be the same at both places? What is that cost?

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

1 Understand the Problem

The **answer** will be the number of games played for which the total cost is the same at both bowling alleys. **List the important information:**

- Game price: Bowl-o-Rama $2.50 Bowling Pinz: $2
- Shoe-rental fee: Bowl-o-Rama $2 Bowling Pinz: $4

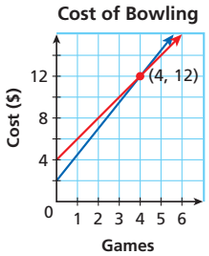
2 Make a Plan

Write a system of equations, one equation to represent the price at each company. Let x be the number of games played and y be the total cost.

	Total cost	is	price per game	times	games	plus	shoe rental.
Bowl-o-Rama	y	=	2.5	•	x	+	2
Bowling Pinz	y	=	2	•	x	+	4

3 Solve

Graph $y = 2.5x + 2$ and $y = 2x + 4$. The lines appear to intersect at $(4, 12)$. So, the cost at both places will be the same for 4 games bowled and that cost will be $12.



4 Look Back

Check $(4, 12)$ using both equations.

Cost of bowling 4 games at Bowl-o-Rama:
$\$2.5(4) + \$2 = 10 + 2 = 12$ ✓

Cost of bowling 4 games at Bowling Pinz:
$\$2(4) + \$4 = 8 + 4 = 12$ ✓

CHECK IT OUT!

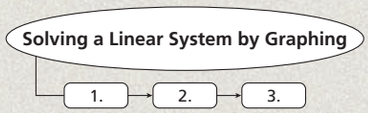
3. Video club A charges $10 for membership and $3 per movie rental. Video club B charges $15 for membership and $2 per movie rental. For how many movie rentals will the cost be the same at both video clubs? What is that cost? **5 movies; $25**

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain how to use a graph to solve a system of linear equations.
2. Explain how to check a solution of a system of linear equations.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write a step for solving a linear system by graphing. More boxes may be added.

Know it! Note



COMMON ERROR ALERT

Students sometimes have difficulty correctly assigning x and y variables in word problems, such as in **Example 3.** Remind them that the value of y cannot be determined unless the value of x is known. In other words, y is dependent on x.

Power Presentations with PowerPoint®

Additional Examples

Example 3

Wren and Jenni are reading the same book. Wren is on page 14 and reads 2 pages every night. Jenni is on page 6 and reads 3 pages every night. After how many nights will they have read the same number of pages? 8 nights How many pages will that be? 30 pages

Also available online

MATHEMATICAL PRACTICES

Questioning Strategies

EXAMPLE 3

- What does the graph show on the left side of the point of intersection?
- What does the graph show on the right side of the point of intersection?

Teaching Tip

Visual Cues In **Example 3** students may benefit from using a table to organize the information.

	Shoe Rental	Game Price
Bowl-o-Rama	$2.00	$2.50
Bowling Pinz	$4.00	$2.00

3 Close

Summarize

Have students graph the equations below to determine whether each ordered pair is the solution of the given system. Have them check their work by substituting the ordered pair into the system of equations.

1. $(0, -2)$; $\begin{cases} y = x - 2 \\ 4y + \frac{1}{2}x = -8 \end{cases}$ yes

2. $(3, 6)$; $\begin{cases} y + 3x = 9 \\ y = 2x \end{cases}$ no

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Locate the point of intersection. The ordered pair is a solution of the system.
2. Substitute the x- and y-values of the ordered pair into all of the equations. If all equations are true, the solution is correct.
3. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**

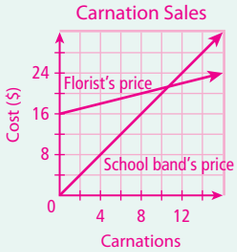
Basic 9–22, 28–30
Average 9–16, 18–22, 26–32
Advanced 9–16, 18–32

Homework Quick Check
Quickly check key concepts.
Exercises: 10, 14, 16, 18, 22

Answers

17a. $\begin{cases} y = 2x \\ y = 16 + 0.50x \end{cases}$

b.



It represents how many carnations need to be sold to break even.

c. No; because the solution is not a whole number of carnations; 11 carnations.

18a. Hats Off: $c = 5h + 50$; Top Stuff: $c = 6h + 25$

b. 25 hats; $175; for graph, see Additional Answers.

5-1 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. Vocabulary Describe a *solution of a system of linear equations.*
an ordered pair that satisfies both equations

SEE EXAMPLE 1

Tell whether the ordered pair is a solution of the given system.

2. $(2, -2)$; $\begin{cases} 3x + y = 4 \\ x - 3y = -4 \end{cases}$ no

3. $(3, -1)$; $\begin{cases} x - 2y = 5 \\ 2x - y = 7 \end{cases}$ yes

4. $(-1, 5)$; $\begin{cases} -x + y = 6 \\ 2x + 3y = 13 \end{cases}$ yes

SEE EXAMPLE 2

Solve each system by graphing. Check your answer.

5. $\begin{cases} y = \frac{1}{2}x \\ y = -x + 3 \end{cases}$ (2, 1)

6. $\begin{cases} y = x - 2 \\ 2x + y = 1 \end{cases}$ (1, −1)

7. $\begin{cases} -2x - 1 = y \\ x + y = 3 \end{cases}$ (−4, 7)

SEE EXAMPLE 3

8. To deliver mulch, Lawn and Garden charges $30 per cubic yard of mulch plus a $30 delivery fee. Yard Depot charges $25 per cubic yard of mulch plus a $55 delivery fee. For how many cubic yards will the cost be the same? What will that cost be?
5 yd^3; $180

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–11	1
12–15	2
16	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Tell whether the ordered pair is a solution of the given system.

9. $(1, -4)$; $\begin{cases} x - 2y = 8 \\ 4x - y = 8 \end{cases}$ no

10. $(-2, 1)$; $\begin{cases} 2x - 3y = -7 \\ 3x + y = -5 \end{cases}$ yes

11. $(5, 2)$; $\begin{cases} 2x + y = 12 \\ -3y - x = -11 \end{cases}$ yes

Solve each system by graphing. Check your answer.

12. $\begin{cases} y = \frac{1}{2}x + 2 \\ y = -x - 1 \end{cases}$ (−2, 1)

13. $\begin{cases} y = x \\ y = -x + 6 \end{cases}$ (3, 3)

14. $\begin{cases} -2x - 1 = y \\ x = -y + 3 \end{cases}$ (−4, 7)

15. $\begin{cases} x + y = 2 \\ y = x - 4 \end{cases}$ (3, −1)

16. Multi-Step Angelo runs 7 miles per week and increases his distance by 1 mile each week. Marc runs 4 miles per week and increases his distance by 2 miles each week. In how many weeks will Angelo and Marc be running the same distance? What will that distance be? 3 weeks; 10 mi

17. School The school band sells carnations on Valentine's Day for $2 each. They buy the carnations from a florist for $0.50 each, plus a $16 delivery charge.

a. Write a system of equations to describe the situation.

b. Graph the system. What does the solution represent?

c. Explain whether the solution shown on the graph makes sense in this situation. If not, give a reasonable solution.

18. a. The Warrior baseball team is selling hats as a fund-raiser. They contacted two companies. Hats Off charges a $50 design fee and $5 per hat. Top Stuff charges a $25 design fee and $6 per hat. Write an equation for each company's pricing.

b. Graph the system of equations from part **a.** For how many hats will the cost be the same? What is that cost?

c. Explain when it is cheaper for the baseball team to use Top Stuff and when it is cheaper to use Hats Off. fewer than 25 hats: Top Stuff; more than 25 hats: Hats Off

Victoria Smith/HMH

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 8, 16–18, 23–25, 31–32

Construct viable arguments and critique the reasoning of others. Exercises 26–27

Use appropriate tools strategically. Exercises 19–22

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READING STRATEGIES

In the example below, the steps show how to set up and solve a system of linear equations.

A teacher needs to rent video equipment. Rent All charges $60 plus $12 for each hour. Rent It Here charges $80 plus $10 per hour. After how many hours is the cost of renting video equipment the same from both companies?

1. Define variables for the unknowns.
   - Let x = number of rental hours
   - Let y = total cost
2. Write a system of linear equations.
   $y = 12x + 60$ (Rent All)
   $y = 10x + 80$ (Rent It Here)
3. Graph the equations on the same grid.
4. Identify the point of intersection.
5. Write your answer as a sentence.
   After 10 hours, the cost of renting video equipment is $180 from both companies.

Derek is going to have a party catered. Good Eats charges $120 plus $10 per person. Food Fare charges $150 plus $8 per person. Complete the following to find the number of people for which the total cost is the same for both catering companies.

1. Define the variables. Let x = number of people. Let y = total cost
2. Write a system of linear equations. $\begin{cases} y = 10x + 120 \\ y = 8x + 150 \end{cases}$
3. Graph the equations.
4. What is the point of intersection? (15, 270)
5. Write your answer. For 15 people, the total cost will be $270 with both catering companies.

RETEACH

You have checked to see if an ordered pair was a solution of an equation. Now you will check to see if an ordered pair is a solution of a system of equations.

Tell whether (1, 9) is a solution of $\begin{cases} x + y = 10 \\ 3x + y = 12 \end{cases}$

Step 1: Substitute (1, 9) into one of the equations.
(1, 9) means that $x = 1$ and $y = 9$.
$x + y \stackrel{?}{=} 10$
$1 + 9 \stackrel{?}{=} 10$
$10 \stackrel{?}{=} 10$ ✓
Solution checks. Continue with Step 2

Step 2: Substitute (1, 9) into the other equation.
$3x + y = 12$
$3(1) + 9 \stackrel{?}{=} 12$
$3 + 9 \stackrel{?}{=} 12$
$12 \stackrel{?}{=} 12$ ✓
The ordered pair makes both equations true. So (1, 9) is a solution of the system.

Tell whether (2, −3) is a solution of $\begin{cases} x + y = 5 \\ 2x + 5y = -11 \end{cases}$

Step 1: Substitute (2, −3) into one of the equations.
$x + y = 5$
$2 + -3 \stackrel{?}{=} 5$
$-1 \stackrel{?}{=} 5$ ×

Stop! There is no need to check the other equation. The ordered pair is not a solution of the system.

Tell whether the ordered pair is a solution of the given system.

1. (0, −4); $\begin{cases} x + 2y = -8 \\ x = 4 + y \end{cases}$ yes
2. (2, 5); $\begin{cases} x + y = 7 \\ 3x + y = 10 \end{cases}$ no
3. (−3, 1); $\begin{cases} 2x + y = 5 \\ x + 3y = -6 \end{cases}$ no
4. (−3, 9); $\begin{cases} y = x + 12 \\ y = -3x \end{cases}$ yes

Graphing Calculator Use a graphing calculator to graph and solve the systems of equations in Exercises 19–22. Round your answer to the nearest tenth.

19. $\begin{cases} y = 4.7x + 2.1 \\ y = 1.6x - 5.4 \end{cases}$ **(−2.4, −9.3)**

20. $\begin{cases} 4.8x + 0.6y = 4 \\ y = -3.2x + 2.7 \end{cases}$ **(0.8, 0.1)**

21. $\begin{cases} y = \frac{5}{4}x - \frac{2}{3} \\ \frac{8}{3}x + y = \frac{5}{9} \end{cases}$ **(0.3, −0.3)**

22. $\begin{cases} y = 6.9x + 12.4 \\ y = -4.1x - 5.3 \end{cases}$ **(−1.6, 1.3)**

LINK

Landscaping

Middleton Place Gardens, South Carolina, are the United States' oldest landscaped gardens. The gardens were established in 1741 and opened to the public in the 1920s.

23. Landscaping The gardeners at Middleton Place Gardens want to plant a total of 45 white and pink hydrangeas in one flower bed. In another flower bed, they want to plant 120 hydrangeas. In this bed, they want 2 times the number of white hydrangeas and 3 times the number of pink hydrangeas as in the first bed. Use a system of equations to find how many white and how many pink hydrangeas the gardeners should buy altogether. **45 white; 120 pink**

24. Fitness Rusty burns 5 Calories per minute swimming and 11 Calories per minute jogging. In the morning, Rusty burns 200 Calories walking and swims for x minutes. In the afternoon, Rusty will jog for x minutes. How many minutes must he jog to burn at least as many Calories y in the afternoon as he did in the morning? Round your answer up to the next whole number of minutes. **34 min**

25. A tree that is 2 feet tall is growing at a rate of 1 foot per year. A 6-foot tall tree is growing at a rate of 0.5 foot per year. In how many years will the trees be the same height? **8 yr**

26. Critical Thinking Write a real-world situation that could be represented by the system $\begin{cases} y = 3x + 10 \\ y = 5x + 20 \end{cases}$.

27. Write About It When you graph a system of linear equations, why does the intersection of the two lines represent the solution of the system?
Every point on a line satisfies the related linear equation. A point that is on both lines (the intersection point) satisfies both equations.

TEST PREP

28. Taxi company A charges $4 plus $0.50 per mile. Taxi company B charges $5 plus $0.25 per mile. Which system best represents this problem?

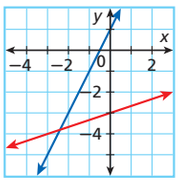
Ⓐ $\begin{cases} y = 4x + 0.5 \\ y = 5x + 0.25 \end{cases}$

Ⓒ $\begin{cases} y = -4x + 0.5 \\ y = -5x + 0.25 \end{cases}$

Ⓑ $\begin{cases} y = 0.5x + 4 \\ y = 0.25x + 5 \end{cases}$

Ⓓ $\begin{cases} y = -0.5x + 4 \\ y = -0.25x + 5 \end{cases}$

29. Which system of equations represents the given graph?



Ⓕ $\begin{cases} y = 2x - 1 \\ y = \frac{1}{3}x + 3 \end{cases}$

Ⓗ $\begin{cases} y = 2x + 1 \\ y = \frac{1}{3}x - 3 \end{cases}$

Ⓖ $\begin{cases} y = -2x + 1 \\ y = 2x - 3 \end{cases}$

Ⓙ $\begin{cases} y = -2x - 1 \\ y = 3x - 3 \end{cases}$

30. Gridded Response Which value of b will make the system $y = 2x + 2$ and $y = 2.5x + b$ intersect at the point (2, 6)? **1**

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COMMON ERROR ALERT

In **Exercises 5–7** and **12–15,** students may get the wrong answer because of inaccurate graphs. Caution students to use a straightedge when drawing lines.

MULTI-STEP TEST PREP **Exercise 18** involves writing a system of equations to compare pricing options. This exercise prepares students for the Multi-Step Test Prep.

Teaching Tip **Inclusion** In **Exercise 23,** encourage students to check that they answered the question asked in the problem.

TEST PREP DOCTOR In **Exercise 28,** choices **C** and **D** can be eliminated because the situation in the problem requires lines with positive slopes.

Choices **G** and **J** can be eliminated in **Exercise 29** because the graph shows two lines with positive slopes.

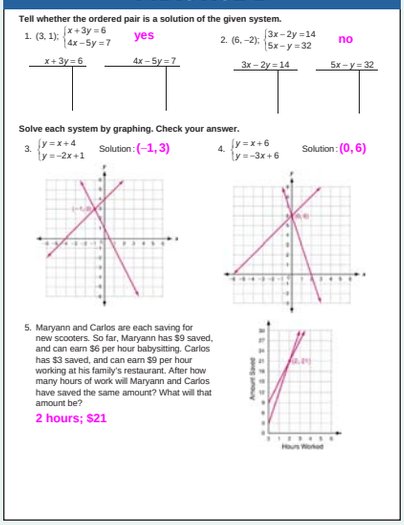
Answers

26. Possible answer: Store A rents carpet cleaners for a fee of $10, plus $3 per day. Store B rents carpet cleaners for a fee of $20, plus $5 per day. For how many days rental will the costs be the same?

PRACTICE A

PRACTICE C

PRACTICE B



Tell whether the ordered pair is a solution of the given system.

1. (3, 1); $\begin{cases} x + 3y = 6 \\ 4x - 5y = 7 \end{cases}$ yes

2. (6, −2); $\begin{cases} 3x - 2y = 14 \\ 5x - y = 32 \end{cases}$ no

$x + 3y = 6$ $4x - 5y = 7$ $3x - 2y = 14$ $5x - y = 32$

Solve each system by graphing. Check your answer.

3. $\begin{cases} y = x + 4 \\ y = -2x + 1 \end{cases}$ Solution: (−1, 3)

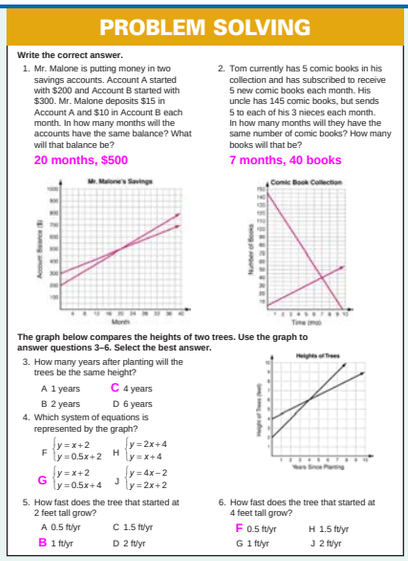
4. $\begin{cases} y = x + 6 \\ y = -3x + 6 \end{cases}$ Solution: (0, 6)

5. Maryann and Carlos are each saving for new scooters. So far, Maryann has $9 saved, and can earn $6 per hour babysitting. Carlos has $3 saved, and can earn $9 per hour working at his family's restaurant. After how many hours of work will Maryann and Carlos have saved the same amount? What will that amount be?
2 hours; $21

Hours Worked

Amount Saved

PROBLEM SOLVING



Write the correct answer.

1. Mr. Malone is putting money in two savings accounts. Account A started with $200 and Account B started with $300. Mr. Malone deposits $15 in Account A and $10 in Account B each month. In how many months will the accounts have the same balance? What will that balance be?
20 months, $500

2. Tom currently has 5 comic books in his collection and has subscribed to receive 5 new comic books each month. His uncle has 145 comic books, but sends 5 to each of his 3 nieces each month. In how many months will they have the same number of comic books? How many books will that be?
7 months, 40 books

Mr. Malone's Savings — Account Balance ($) — Month

Comic Book Collection — Number of Books — Time (mo)

The graph below compares the heights of two trees. Use the graph to answer questions 3–6. Select the best answer.

3. How many years after planting will the trees be the same height?
A 1 years C 4 years
B 2 years D 6 years

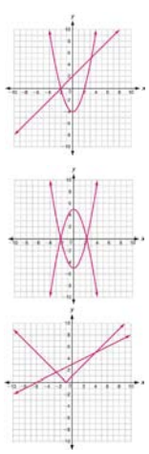
4. Which system of equations is represented by the graph?
F $\begin{cases} y = x + 2 \\ y = 0.5x + 2 \end{cases}$ H $\begin{cases} y = 2x + 4 \\ y = x + 4 \end{cases}$
G $\begin{cases} y = x + 2 \\ y = 0.5x + 4 \end{cases}$ J $\begin{cases} y = 4x - 2 \\ y = 2x + 2 \end{cases}$

Heights of Trees — Height of Trees (feet) — Years Since Planting

5. How fast does the tree that started at 2 feet tall grow?
A 0.5 ft/yr C 1.5 ft/yr
B 1 ft/yr D 2 ft/yr

6. How fast does the tree that started at 4 feet tall grow?
F 0.5 ft/yr H 1.5 ft/yr
G 1 ft/yr J 2 ft/yr

CHALLENGE



Sometimes one or both equations in a system will be non-linear. The solutions to these systems will still be where the graphs intersect. This can happen in more than one place. Recall that you can graph any equation by generating and plotting ordered pairs.

Solve each system by graphing.

1. $\begin{cases} y = x^2 - 4 \\ y = x + 2 \end{cases}$

(−2, 0) and (3, 5)

2. $\begin{cases} y = -x^2 + 5 \\ y = x^2 - 5 \end{cases}$

about (−2.3, 0) and about (2.3, 0)

3. $\begin{cases} y = \lvert x + 1 \rvert \\ y = \frac{1}{2}x + 3 \end{cases}$

about (−2.7, 1.7) and (4, 5)

CHALLENGE AND EXTEND

31. Entertainment If the pattern in the table continues, in what month will the number of sales of VCRs and DVD players be the same? What will that number be? **month 11; 400**

Total Number Sold				
Month	1	2	3	4
VCRs	500	490	480	470
DVD Players	250	265	280	295

32. Long Distance Inc. charges a $1.45 connection charge and $0.03 per minute. Far Away Calls charges a $1.52 connection charge and $0.02 per minute.

a. For how many minutes will a call cost the same from both companies? What is that cost? **7 min; $1.66**

b. When is it better to call using Long Distance Inc.? Far Away Calls? Explain.

c. What if...? Long Distance Inc. raised its connection charge to $1.50 and Far Away Calls decreased its connection charge by 2 cents. How will this affect the graphs? Now which company is better to use for calling long distance? Why?

32b. It is better to use Long Distance Inc. if the call is under 7 minutes. If the call is over 7 minutes, it is better to use Far Away Calls.

c. The graphs will intersect the *y*-axis at 1.5. Far Away Calls is cheaper because it costs less per minute.

Career Path

Learn It Online
Career Resources Online

Ethan Reynolds
Applied Sciences major

Q: What math classes did you take in high school?
A: Career Math, Algebra, and Geometry

Q: What are you studying and what math classes have you taken?
A: I am really interested in aviation. I am taking Statistics and Trigonometry. Next year I will take Calculus.

Q: How is math used in aviation?
A: I use math to interpret aeronautical charts. I also perform calculations involving wind movements, aircraft weight and balance, and fuel consumption. These skills are necessary for planning and executing safe air flights.

Q: What are your future plans?
A: I could work as a commercial or corporate pilot or even as a flight instructor. I could also work toward a bachelor's degree in aviation management, air traffic control, aviation electronics, aviation maintenance, or aviation computer science.

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Journal

Have students explain how to determine whether an ordered pair is the solution of a system of equations.

ALTERNATIVE ASSESSMENT

Have students choose a point on the coordinate grid, draw two lines through the point, and write the equations for those lines. Then have students show algebraically that the coordinates of the point of intersection are a solution of both equations.

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5-1 Lesson Quiz

Tell whether the ordered pair is a solution of the given system.

1. $(-3, 1)$; $\begin{cases} y = -\frac{1}{3}x \\ 4y + 12 = 3x \end{cases}$ no

2. $(2, -4)$; $\begin{cases} y - 2x = -8 \\ 4y + 13x = 10 \end{cases}$ yes

Solve the system by graphing.

3. $\begin{cases} y + 2x = 9 \\ y = 4x - 3 \end{cases}$ $(2, 5)$

4. Joy has 5 collectible stamps and will buy 2 more each month. Ronald has 25 collectible stamps and will sell 3 each month. After how many months will they have the same number of stamps? 4 months How many will that be? 13 stamps

Also available online

5-2 Algebra LAB

Model Systems of Linear Equations

You can use algebra tiles to model and solve some systems of linear equations.

Use with Solving Systems by Substitution

KEY

+ = 1

− = −1

+ = x

− = $-x$

REMEMBER

When two expressions are equal, you can substitute one for the other in any expression or equation.

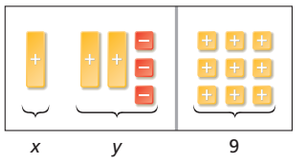
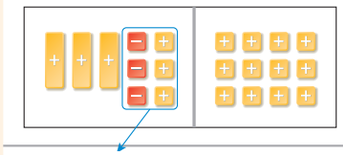
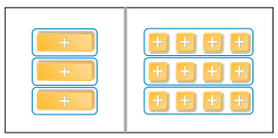
Learn It Online
Lab Resources Online

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Activity

Use algebra tiles to model and solve $\begin{cases} y = 2x - 3 \\ x + y = 9 \end{cases}$.

MODEL	ALGEBRA
 *The first equation is solved for y. Model the second equation, $x + y = 9$, by substituting $2x - 3$ for y.*	$x + y = 9$ $x + (2x - 3) = 9$ $3x - 3 = 9$
 *Add 3 yellow tiles on both sides of the mat. This represents adding 3 to both sides of the equation. Remove zero pairs.*	$3x - 3 = 9$ $+3 \quad +3$ $3x = 12$
 *Divide each side into 3 equal groups. Align one x-tile with each group on the right side. One x-tile is equivalent to 4 yellow tiles. x = 4*	$\frac{3x}{3} = \frac{12}{3}$ $x = 4$

To solve for y, substitute 4 for x in one of the equations:

$$\begin{aligned} y &= 2x - 3 \\ &= 2(4) - 3 \\ &= 5 \end{aligned}$$

The solution is (4, 5).

Try This

Model and solve each system of equations.

1. $\begin{cases} y = x + 3 \\ 2x + y = 6 \end{cases}$
2. $\begin{cases} 2x + 3 = y \\ x + y = 6 \end{cases}$
3. $\begin{cases} 2x + 3y = 1 \\ x = -1 - y \end{cases}$
4. $\begin{cases} y = x + 1 \\ 2x - y = -5 \end{cases}$

1. (1, 4)
2. (1, 5)
3. (−4, 3)
4. (−4, −3)

Organizer

Use with the lesson Solving Systems by Substitution

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use algebra tiles to model and solve systems of linear equations.

Materials: algebra tiles

Online Edition
Algebra Tiles

Countdown

Lab Resources

Algebra Activities
Lab Recording Sheet

Teach

Discuss

You can represent either x or y with the variable tile. Make sure the variable tile represents the same variable throughout the same system. Algebra tiles can be found in the Manipulatives Kit (MK).

Close

Key Concept

You can solve a system of linear equations by modeling one variable in terms of the other, substituting that value into the other equation, and solving.

Assessment

Journal Have students explain how to use algebra tiles to solve one of the systems in the *Try This* problems.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Learn It Online
State Resources Online

5-2 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Solve systems of linear equations in two variables by substitution.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Solve each equation for *x*.

1. $y = x + 3$ $x = y - 3$

2. $y = 3x - 4$ $x = \frac{y + 4}{3}$

Simplify each expression.

3. $2(x - 5)$ $2x - 10$

4. $12 - 3(x + 1)$ $9 - 3x$

Evaluate each expression for the given value of *x*.

5. $\frac{2}{3}x + 8$ for $x = 6$ 12

6. $3(x - 7)$ for $x = 10$ 9

Also available online

Math Humor

Q: Why did the chef throw the unsolved system of equations out of the restaurant?

A: The menu said, "No substitution."

COMMON CORE

CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
CC.9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.*

Learn It Online
State Resources Online

COMMON CORE

5-2

Solving Systems by Substitution

CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. *Also* **CC.9-12.A.CED.3***

Objective
Solve systems of linear equations in two variables by substitution.

Why learn this?
You can solve systems of equations to help select the best value among high-speed Internet providers. (See Example 3.)

Off the Mark by Mark Parisi. Cartoon copyrighted by Mark Parisi, printed with permission.

Sometimes it is difficult to identify the exact solution to a system by graphing. In this case, you can use a method called *substitution.*

The goal when using substitution is to reduce the system to one equation that has only one variable. Then you can solve this equation, and substitute into an original equation to find the value of the other variable.

Know it! Note

Solving Systems of Equations by Substitution
Step 1 Solve for one variable in at least one equation, if necessary.
Step 2 Substitute the resulting expression into the other equation.
Step 3 Solve that equation to get the value of the first variable.
Step 4 Substitute that value into one of the original equations and solve.
Step 5 Write the values from Steps 3 and 4 as an ordered pair, (x, y), and check.

EXAMPLE 1 Solving a System of Linear Equations by Substitution

Solve each system by substitution.

A $\begin{cases} y = 2x \\ y = x + 5 \end{cases}$

Step 1 $y = 2x$
$y = x + 5$ *Both equations are solved for y.*

Step 2 $y = x + 5$
$2x = x + 5$ *Substitute 2x for y in the second equation.*

Step 3 $-x \quad -x$
$x = 5$ *Solve for x.*

Step 4 $y = 2x$ *Write one of the original equations.*
$y = 2(5)$ *Substitute 5 for x.*
$y = 10$

Step 5 $(5, 10)$ *Write the solution as an ordered pair.*

Check Substitute $(5, 10)$ into both equations in the system.

$y = 2x$	
10	$2(5)$
10	10 ✓

$y = x + 5$	
10	$5 + 5$
10	10 ✓

Helpful Hint
You can substitute the value of one variable into *either* of the original equations to find the value of the other variable.

1 Introduce

EXPLORATION

Solving Systems by Substitution

You can sometimes use the equations in a system of equations to write a new equation.

1. Consider the two equations shown here. What new equation can you write? Explain. $a = b$, $a = c$
2. Use the idea from Step 1 to write a new equation based on each of the following systems.
   a. $\begin{cases} y = 2x + 1 \\ y = 3x - 4 \end{cases}$ $2x + 1 = 3x - 4$
   b. $\begin{cases} x = 3y + 2 \\ x = 5y + 4 \end{cases}$ $3y + 2 = 5y + 4$
   c. $\begin{cases} m = -4t + 2 \\ m = t - 8 \end{cases}$ $-4t + 2 = t - 8$
   d. $\begin{cases} 3y = 2x + 7 \\ 3y = -x + 4 \end{cases}$ $2x + 7 = -x + 4$

THINK AND DISCUSS

3. **Describe** what you notice about all of the equations you wrote in Step 2. (*Hint:* How many variables are in each equation?) Each equation has only one variable.
4. **Discuss** whether it would be possible to use the above method to write a new equation based on this system: $\begin{cases} y = 2x + 6 \\ y + 1 = 4x \end{cases}$.

Motivate

In sports, coaches often substitute one player for another who plays the same position. Ask students to suggest other situations in which substitutions are made. Tell them that in a system of equations, you also can make substitutions. You can replace a variable in an equation with an equivalent expression.

Explorations and answers are provided in your online resources.

Solve each system by substitution.

B $\begin{cases} 2x + y = 5 \\ y = x - 4 \end{cases}$

Step 1 $y = x - 4$ *The second equation is solved for y.*

Step 2 $2x + y = 5$

$2x + (x - 4) = 5$ *Substitute x − 4 for y in the first equation.*

Step 3 $3x - 4 = 5$ *Simplify. Then solve for x.*

$+4 \quad +4$ *Add 4 to both sides.*

$3x = 9$

$\frac{3x}{3} = \frac{9}{3}$ *Divide both sides by 3.*

$x = 3$

Step 4 $y = x - 4$ *Write one of the original equations.*

$y = 3 - 4$ *Substitute 3 for x.*

$y = -1$

Step 5 $(3, -1)$ *Write the solution as an ordered pair.*

C $\begin{cases} x + 4y = 6 \\ x + y = 3 \end{cases}$

Step 1 $x + 4y = 6$ *Solve the first equation for x by subtracting 4y from both sides.*

$-4y \quad -4y$

$x = 6 - 4y$

Step 2 $x + y = 3$

$(6 - 4y) + y = 3$ *Substitute 6 − 4y for x in the second equation.*

Step 3 $6 - 3y = 3$ *Simplify. Then solve for y.*

$-6 \quad -6$ *Subtract 6 from both sides.*

$-3y = -3$

$\frac{-3y}{-3} = \frac{-3}{-3}$ *Divide both sides by −3.*

$y = 1$

Step 4 $x + y = 3$ *Write one of the original equations.*

$x + 1 = 3$ *Substitute 1 for y.*

$-1 \quad -1$ *Subtract 1 from both sides.*

$x = 2$

Step 5 $(2, 1)$ *Write the solution as an ordered pair.*

Helpful Hint

Sometimes neither equation is solved for a variable. You can begin by solving either equation for either *x* or *y*.

CHECK IT OUT! Solve each system by substitution.

1a. $\begin{cases} y = x + 3 \\ y = 2x + 5 \end{cases}$ $(-2, 1)$

1b. $\begin{cases} x = 2y - 4 \\ x + 8y = 16 \end{cases}$ $(0, 2)$

1c. $\begin{cases} 2x + y = -4 \\ x + y = -7 \end{cases}$ $(3, -10)$

Sometimes you substitute an expression for a variable that has a coefficient. When solving for the second variable in this situation, you can use the Distributive Property.

COMMON ERROR ALERT

Students often think they have solved a system after finding the value of only one variable. Remind them that the solution is an ordered pair.

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Additional Examples

Example 1

Solve each system by substitution.

A. $\begin{cases} y = 3x \\ y = x - 2 \end{cases}$ $(-1, -3)$

B. $\begin{cases} y = x + 1 \\ 4x + y = 6 \end{cases}$ $(1, 2)$

C. $\begin{cases} x + 2y = -1 \\ x - y = 5 \end{cases}$ $(3, -2)$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- How is substituting an expression for a variable similar to substituting a number for a variable? How is it different?
- In **Example 1C,** how do you know which variable to isolate?

Teaching Tip **Inclusion** Point out that in **Example 1A** both equations are written in slope-intercept form, so isolating a variable is unnecessary. In **Example 1B,** one equation is in slope-intercept form. The expression containing *x* from that equation can be substituted into the other equation. In **Example 1C,** neither equation is written in slope-intercept form. First, a variable must be isolated in one or both equations.

2 Teach

Guided Instruction

Review how to apply the Distributive Property and combine like terms to simplify an expression before **Example 2.** Have students practice solving by substitution when at least one equation is in slope-intercept form before having them solve systems where they first need to rewrite an equation. Remind students to check their solutions by substituting the values back into both equations or by graphing both lines and locating their intersection.

Reaching All Learners *Through Cooperative Learning*

Have students work in pairs, and give them the system $\begin{cases} 2x - 4y = 10 \\ y + 5x = 3 \end{cases}$. Have one student solve the first equation for *y* and the other solve the second equation for *y*. Then have both continue independently to solve by using substitution. Each should arrive at the solution $(1, -2)$. Have them compare their work and discuss which substitution is more efficient.

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Additional Examples

Example 2

Solve $\begin{cases} y + 6x = 11 \\ 3x + 2y = -5 \end{cases}$ by substitution. $(3, -7)$

Also available online

Questioning Strategies

EXAMPLE 2

- When do you need to use the Distributive Property?
- Why is it important to put the expression you are substituting for a variable in parentheses?

Multiple Representations

Allowing students to check their solutions by graphing on a calculator and finding points of intersection will provide a quick check as well as emphasize that there are multiple representations of a system of linear equations.

EXAMPLE 2 Using the Distributive Property

Solve $\begin{cases} 4y - 5x = 9 \\ x - 4y = 11 \end{cases}$ by substitution.

Step 1 $x - 4y = 11$

$+4y \quad +4y$

$x = 4y + 11$

Solve the second equation for x by adding 4y to each side.

Step 2 $4y - 5x = 9$

$4y - 5(4y + 11) = 9$

Substitute 4y + 11 for x in the first equation.

Step 3 $4y - 5(4y) - 5(11) = 9$

$4y - 20y - 55 = 9$

Distribute −5 to the expression in parentheses. Simplify. Solve for y.

$-16y - 55 = 9$

$+55 \quad +55$

Add 55 to both sides.

$-16y = 64$

$\frac{-16y}{-16} = \frac{64}{-16}$

Divide both sides by −16.

$y = -4$

Step 4 $x - 4y = 11$

Write one of the original equations.

$x - 4(-4) = 11$

Substitute −4 for y.

$x + 16 = 11$

Simplify.

$-16 \quad -16$

Subtract 16 from both sides.

$x = -5$

Step 5 $(-5, -4)$

Write the solution as an ordered pair.

Caution!

When you solve one equation for a variable, you must substitute the value or expression into the *other* original equation, not the one that has just been solved.

CHECK IT OUT! **2.** Solve $\begin{cases} -2x + y = 8 \\ 3x + 2y = 9 \end{cases}$ by substitution. $(-1, 6)$

Student to Student

Solving Systems by Substitution

Erika Chu
Terrell High School

I always look for a variable with a coefficient of 1 or −1 when deciding which equation to solve for x or y.

For the system

$\begin{cases} 2x + y = 14 \\ -3x + 4y = -10 \end{cases}$

I would solve the first equation for y because it has a coefficient of 1.

$2x + y = 14$

$y = -2x + 14$

Then I use substitution to find the values of x and y.

$-3x + 4y = -10$

$-3x + 4(-2x + 14) = -10$

$-3x + (-8x) + 56 = -10$

$-11x + 56 = -10$

$-11x = -66$

$x = 6$

$y = -2x + 14$

$y = -2(6) + 14 = 2$

The solution is (6, 2).

© Michael Pole/SuperStock

EXAMPLE 3 Consumer Economics Application

One high-speed Internet provider has a $50 setup fee and costs $30 per month. Another provider has no setup fee and costs $40 per month.

a. In how many months will both providers cost the same? What will that cost be?

Write an equation for each option. Let t represent the total amount paid and m represent the number of months.

	Total paid	is	setup fee	plus	cost per month	times	months.
Option 1	t	=	50	+	30	•	m
Option 2	t	=	0	+	40	•	m

Step 1 $t = 50 + 30m$
$t = 40m$ — *Both equations are solved for t.*

Step 2 $50 + 30m = 40m$ — *Substitute 50 + 30m for t in the second equation.*

Step 3 $-30m \quad -30m$
$50 = 10m$ — *Solve for m. Subtract 30m from both sides.*

$\frac{50}{10} = \frac{10m}{10}$ — *Divide both sides by 10.*

$5 = m$

Step 4 $t = 40m$ — *Write one of the original equations.*
$= 40(5)$ — *Substitute 5 for m.*
$= 200$

Step 5 $(5, 200)$ — *Write the solution as an ordered pair.*

In 5 months, the total cost for each option will be the same—$200.

b. If you plan to cancel in 1 year, which is the cheaper provider? Explain.

Option 1: $t = 50 + 30(12) = 410$ Option 2: $t = 40(12) = 480$

Option 1 is cheaper.

CHECK IT OUT!

3. One cable television provider has a $60 setup fee and charges $80 per month, and another provider has a $160 equipment fee and charges $70 per month.

a. In how many months will the cost be the same? What will that cost be?

b. If you plan to move in 6 months, which is the cheaper option? Explain.

3a. 10 months; $860

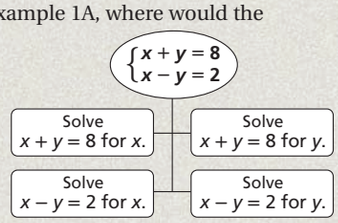
3b. The first option; the first option is cheaper for the first 9 months; the second option is cheaper after 10 months.

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. If you graphed the equations in Example 1A, where would the lines intersect?

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, solve the system by substitution using the first step given. Show that each method gives the same solution.



Know it! Note

3 Close

Summarize

Have students supply the steps they would need to take to solve the system $\begin{cases} 4x - y = 8 \\ 2x + 3y = 4 \end{cases}$ by substitution. Then have them solve it. Possible answer: Solve the first equation for y, then substitute that expression for y in the second equation. Solve for x. Substitute the value for x into the first equation and solve for y. The solution is $(2, 0)$.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. at the point (5, 10)
2. See Additional Answers.

COMMON ERROR ALERT

Students may "lose" a negative sign when they substitute an expression for a variable. Encourage students to write clearly, even when solving problems on scratch paper. A poorly written minus sign can easily be mistaken for a stray pencil mark.

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Additional Examples

Example 3

Jenna is deciding between two cell-phone plans. The first plan has a $50 sign-up fee and costs $20 per month. The second plan has a $30 sign-up fee and costs $25 per month. After how many months will the total costs be the same? What will the cost be? If Jenna has to sign a one-year contract, which plan will be cheaper? Explain. 4 months; $130; Jenna should choose the first plan because it costs $290 for 1 year, and the second plan costs $330.

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 3

- Why can you set the expressions for each option/plan equal to each other?

Teaching Tip **Visual** In **Example 3,** students may benefit from using a table to organize the information.

	Setup Fee	Monthly Rate
Provider 1	$50	$30
Provider 2	$0	$40

5-2 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**

Basic 8–30, 34, 36–37
Average 8–17, 18–30 even, 32–38
Advanced 8–30 even, 31–41

Homework Quick Check
Quickly check key concepts.
Exercises: 10, 12, 16, 20, 26, 30

GUIDED PRACTICE

Solve each system by substitution.

SEE EXAMPLE 1

1. $\begin{cases} y = 5x - 10 \\ y = 3x + 8 \end{cases}$ (9, 35)

2. $\begin{cases} 3x + y = 2 \\ 4x + y = 20 \end{cases}$ (18, −52)

3. $\begin{cases} y = x + 5 \\ 4x + y = 20 \end{cases}$ (3, 8)

SEE EXAMPLE 2

4. $\begin{cases} x - 2y = 10 \\ \frac{1}{2}x - 2y = 4 \end{cases}$ (12, 1)

5. $\begin{cases} y - 4x = 3 \\ 2x - 3y = 21 \end{cases}$ (−3, −9)

6. $\begin{cases} x = y - 8 \\ -x - y = 0 \end{cases}$ (−4, 4)

SEE EXAMPLE 3

7. **Consumer Economics** The Strauss family is deciding between two lawn-care services. Green Lawn charges a $49 startup fee, plus $29 per month. Grass Team charges a $25 startup fee, plus $37 per month.

   a. In how many months will both lawn-care services cost the same? What will that cost be? **3 months; $136**

   b. If the family will use the service for only 6 months, which is the better option? Explain. **Green Lawn; for 6 months, Green Lawn's service costs only $223, while Grass Team's costs $247.**

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8–10	1
11–16	2
17	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each system by substitution.

8. $\begin{cases} y = x + 3 \\ y = 2x + 4 \end{cases}$ (−1, 2)

9. $\begin{cases} y = 2x + 10 \\ y = -2x - 6 \end{cases}$ (−4, 2)

10. $\begin{cases} x + 2y = 8 \\ x + 3y = 12 \end{cases}$ (0, 4)

11. $\begin{cases} 2x + 2y = 2 \\ -4x + 4y = 12 \end{cases}$ (−1, 2)

12. $\begin{cases} y = 0.5x + 2 \\ -y = -2x + 4 \end{cases}$ (4, 4)

13. $\begin{cases} -x + y = 4 \\ 3x - 2y = -7 \end{cases}$ (1, 5)

14. $\begin{cases} 3x + y = -8 \\ -2x - y = 6 \end{cases}$ (−2 −2)

15. $\begin{cases} x + 2y = -1 \\ 4x - 4y = 20 \end{cases}$ (3, −2)

16. $\begin{cases} 4x = y - 1 \\ 6x - 2y = -3 \end{cases}$ $\left(\frac{1}{2}, 3\right)$

17. **Recreation** Casey wants to buy a gym membership. One gym has a $150 joining fee and costs $35 per month. Another gym has no joining fee and costs $60 per month.

   a. In how many months will both gym memberships cost the same? What will that cost be? **6 months; $360**

   b. If Casey plans to cancel in 5 months, which is the better option for him? Explain.

the second option; for 5 months, it costs only $300, while the other option costs $325.

Solve each system by substitution. Check your answer.

18. $\begin{cases} x = 5 \\ x + y = 8 \end{cases}$ (5, 3)

19. $\begin{cases} y = -3x + 4 \\ x = 2y + 6 \end{cases}$ (2, −2)

20. $\begin{cases} 3x - y = 11 \\ 5y - 7x = 1 \end{cases}$ (7, 10)

21. $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 6 \\ x - y = 2 \end{cases}$ (8, 6)

22. $\begin{cases} x = 7 - 2y \\ 2x + y = 5 \end{cases}$ (1, 3)

23. $\begin{cases} y = 1.2x - 4 \\ 2.2x + 5 = y \end{cases}$ (−9, −14.8)

24. $\begin{cases} x + y = 50 \\ x = 2y - 43 \end{cases}$; 19 and 31

24. The sum of two numbers is 50. The first number is 43 less than twice the second number. Write and solve a system of equations to find the two numbers.

25. **Money** A jar contains n nickels and d dimes. There are 20 coins in the jar, and the total value of the coins is $1.40. How many nickels and how many dimes are in the jar? (*Hint:* Nickels are worth $0.05 and dimes are worth $0.10.) **12 nickels; 8 dimes**

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 7, 17, 24–31, 34, 38

Construct viable arguments and critique the reasoning of others. Exercises 32–33

Attend to precision. Exercise 35

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State Resources Online

READING STRATEGIES

Use the sequence chain below to guide you in solving systems of linear equations by the method of substitution.

Step 1: Isolate one variable in one equation.
Step 2: Substitute the expression for that variable into the other equation.
Step 3: Solve for the variable remaining in that equation.
Sequence Chain: *Solving Systems by Substitution*
Step 4: Substitute the value for that variable into either original equation and solve for the other variable.
Step 5: Write the found values for x and y as an ordered pair (x, y).
Step 6: Check your answer by substituting (x, y) into both equations.

Answer each question.

1. Perform Step 1 for $\begin{cases} x+y=5 \\ -3x+5=\frac{1}{2}y \end{cases}$ by solving the first equation for y.
   $y = 5 - x$ or $y = -x + 5$
2. To solve the system $\begin{cases} y=x+2 \\ 2x+y=6 \end{cases}$ by substitution, you can start at Step 2. Explain why.
   The first equation is already solved for y.
3. Why does Step 6 specify substituting into *both* equations?
   The solution of a system must satisfy both equations.

Solve each system of equations by substitution.

4. $\begin{cases} 2x=3y \\ y=x-2 \end{cases}$ (6, 4)
5. $\begin{cases} x+y=2 \\ -x=2y-7 \end{cases}$ (−3, 5)

RETEACH

You can use substitution to solve a system of equations if one of the equations is already solved for a variable.

Solve $\begin{cases} y=x+2 \\ 3x+y=10 \end{cases}$

Step 1: Choose the equation to use as the substitute.
Use the first equation $y = x + 2$ because it is already solved for a variable.

Step 2: Solve by substitution.
$x + 2$
$3x + y = 10$
$3x + (x + 2) = 10$ *Substitute $x + 2$ for y.*
$4x + 2 = 10$ *Combine like terms.*
$-2 \quad -2$
$4x = 8$
$\frac{4x}{4} = \frac{8}{4}$
$x = 2$

Step 3: Now substitute $x = 2$ back into one of the original equations to find the value of y.
$y = x + 2$
$y = 2 + 2$
$y = 4$
The solution is (2, 4).

Check:
Substitute (2, 4) into both equations.
$y = x + 2$ | $3x + y = 10$
$4 \stackrel{?}{=} 2 + 2$ | $3(2) + 4 \stackrel{?}{=} 10$
$4 \stackrel{?}{=} 4$ ✓ | $6 + 4 \stackrel{?}{=} 10$
$10 \stackrel{?}{=} 10$ ✓

Solve each system by substitution. Check your answer.

1. $\begin{cases} x=y-1 \\ x+2y=8 \end{cases}$ (2, 3)
2. $\begin{cases} y=x+2 \\ y=2x-5 \end{cases}$ (7, 9)
3. $\begin{cases} y=x+5 \\ 3x+y=-11 \end{cases}$ (−4, 1)
4. $\begin{cases} x=y+10 \\ x=2y+3 \end{cases}$ (17, 7)

26. Multi-Step Use the receipts below to write and solve a system of equations to find the cost of a large popcorn and the cost of a small drink.

$\begin{cases} 3p + 2d = 21 \\ 2p + 4d = 22 \end{cases}$; popcorn: \$5; drink: \$3

27. $\begin{cases} x + y = 1000 \\ 0.05x + 0.06y = 58 \end{cases}$; \$200 at 5%; \$800 at 6%

27. Finance Helene invested a total of \$1000 in two simple-interest bank accounts. One account paid 5% annual interest; the other paid 6% annual interest. The total amount of interest she earned after one year was \$58. Write and solve a system of equations to find the amount invested in each account. (*Hint:* Change the interest rates into decimals first.)

Geometry **Two angles whose measures have a sum of 90° are called complementary angles. For Exercises 28–30, x and y represent the measures of complementary angles. Use this information and the equation given in each exercise to find the measure of each angle.**

28. $y = 4x - 10$ $x = 20°$; $y = 70°$

29. $x = 2y$ $x = 60°$; $y = 30°$

30. $y = 2(x - 15)$ $x = 40°$; $y = 50°$

31. Aviation With a headwind, a small plane can fly 240 miles in 3 hours. With a tailwind, the plane can fly the same distance in 2 hours. Follow the steps below to find the rates of the plane and wind.

a. Copy and complete the table. Let p be the rate of the plane and w be the rate of the wind.

	Rate	•	Time	=	Distance
With Headwind	$p - w$	•	3	=	240
With Tailwind	$p + w$	•	2	=	240

31b. $\begin{cases} 3p - 3w = 240 \\ 2p + 2w = 240 \end{cases}$

31c. plane: 100 mi/h; wind: 20 mi/h

b. Use the information in each row to write a system of equations.

c. Solve the system of equations to find the rates of the plane and wind.

32. Write About It Explain how to solve a system of equations by substitution.

33. Critical Thinking Explain the connection between the solution of a system solved by graphing and the solution of the same system solved by substitution.

34. At the school store, Juanita bought 2 books and a backpack for a total of \$26 before tax. Each book cost \$8 less than the backpack.

a. Write a system of equations that can be used to find the price of each book and the price of the backpack.

b. Solve this system by substitution.

c. Solve this system by graphing. Discuss advantages and disadvantages of solving by substitution and solving by graphing.

Victoria Smith/HMH

Teaching Tip **Reading Math** For **Exercise 31,** help students understand the terms *headwind* and *tailwind*. A headwind blows in the direction opposite the course of the plane. A tailwind blows in the same direction as the course of the plane.

ENGLISH LANGUAGE LEARNERS

MULTI-STEP TEST PREP **Exercise 34** involves solving a system of equations by at least two different methods to find prices. This exercise prepares students for the Multi-Step Test Prep.

Answers

32. Possible answer: Solve one of the equations for either x or y. Then substitute the expression equal to x or y into the other equation. This creates a one-variable equation that can be solved. When you get the value of one variable, substitute it into one of the original equations and solve to find the value of the other variable.

33. The solution of a system solved by graphing is the same as the solution of a system solved by substitution.

34a. $\begin{cases} 2x + y = 26 \\ x = y - 8 \end{cases}$

b. book: \$6; backpack: \$14

c. Possible answer: Substitution works well since x is already isolated. Graphing requires solving both equations for y.

PRACTICE A

PRACTICE C

PRACTICE B

Solve each system by substitution. Check your answer.

1. $\begin{cases} y = x - 2 \\ y = 4x + 1 \end{cases}$ (–1, –3)
2. $\begin{cases} y = x - 4 \\ y = -x + 2 \end{cases}$ (3, –1)
3. $\begin{cases} y = 3x + 1 \\ y = 5x - 3 \end{cases}$ (2, 7)
4. $\begin{cases} 2x - y = 6 \\ x + y = -3 \end{cases}$ (1, –4)
5. $\begin{cases} 2x + y = 8 \\ y = x - 7 \end{cases}$ (5, –2)
6. $\begin{cases} 2x + 3y = 0 \\ x + 2y = -1 \end{cases}$ (3, –2)
7. $\begin{cases} 3x - 2y = 7 \\ x + 3y = -5 \end{cases}$ (1, –2)
8. $\begin{cases} -2x + y = 0 \\ 5x + 3y = -11 \end{cases}$ (–1, –2)
9. $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 5 \\ \frac{1}{4}x + y = 10 \end{cases}$ (4, 9)

Write a system of equations to represent the situation. Then, solve the system by substitution.

10. The length of a rectangle is 3 more than its width. The perimeter of the rectangle is 58 cm. What are the rectangle's dimensions?

$\begin{cases} l = w + 3 \\ 2l = 2w + 58 \end{cases}$; 13 cm by 16 cm

11. Carla and Benicio work in a men's clothing store. They earn commission from each suit and each pair of shoes they sell. For selling 3 suits and one pair of shoes, Carla has earned \$47 in commission. For selling 7 suits and 2 pairs of shoes, Benicio has earned \$107 in commission. How much do the salespeople earn for the sale of a suit? for the sale of a pair of shoes?

$\begin{cases} 3s + 1p = 47 \\ 7s + 2p = 107 \end{cases}$; suit: \$13; pair of shoes: \$8

PROBLEM SOLVING

Write the correct answer.

1. Maribel has \$1.25 in her pocket. The money is in quarters and dimes. There are a total of 8 coins. How many quarters and dimes does Maribel have in her pocket?
   3 quarters, 5 dimes
2. Fabulously Fit offers memberships for \$35 per month plus a \$50 enrollment fee. The Fitness Studio offers memberships for \$40 per month plus a \$35 enrollment fee. In how many months will the fitness clubs cost the same? What will the cost be?
   3 months; \$155
3. Vong grilled 21 burgers at a block party. He grilled the same number of pounds of turkey burgers as hamburgers. Each turkey burger weighed $\frac{1}{4}$ pound and each hamburger weighed $\frac{1}{3}$ pound. How many of each did Vong grill?
   12 turkey burgers, 9 beef hamburgers
4. Kate bought 3 used CDs and 1 used DVD at the bookstore. Her friend Joel bought 2 used CDs and 2 used DVDs at the same store. If Kate spent \$20 and Joel spent \$22, determine the cost of a used CD and a used DVD.
   used CD \$4.50, used DVD \$6.50

Use the chart below to answer questions 5–8. Select the best answer.
The chart compares the quotes that the Masons received from four different flooring contractors to tear out and replace a floor.

Contractor	Cost to tear out old floor	Cost of new floor per square foot
Smith & Son	\$250	\$8.00
V.I.P. Inc.	\$350	\$7.75
Dad's Floors	\$150	\$8.00
Floorshop	\$300	\$8.25

5. Which expression shows the total cost if the work is done by Dad's Floors?
   A $8 + 150x$ C $150(8x)$
   B $150 + 8x$ D $158x$
6. How many square feet would the Masons need to have installed to make the total cost of V.I.P. Inc. the same as the total cost of Floorshop?
   F 10 sq ft **H** 100 sq ft
   G 200 sq ft J 350 sq ft
7. When the total costs of V.I.P. Inc. and Floorshop are the same, what is the total cost?
   A \$1125.00 C \$1950.00
   B \$1900.00 D \$3187.50
8. How many square feet would the Masons need to have installed to make the total cost of Smith & Son the same as the total cost of V.I.P. Inc.?
   F 80 sq ft **H** 400 sq ft
   G 100 sq ft J 1000 sq ft

CHALLENGE

To solve a system with two variables, you must have two equations. To solve a system in three variables, you must have three equations.

Use substitution to find the values of x, y, and z.

1. $\begin{cases} 2x + 2y + 2z = 4 \\ y + 2z = 1 \\ z = 2 \end{cases}$ $x = 3$; $y = -1$, $z = 4$
2. $\begin{cases} 4x + 2y + 2z = 9 \\ 2y + 2z = 3 \\ z = 3 \end{cases}$ $x = -1$; $y = 4$, $z = -3$
3. $\begin{cases} 2x + 2y + 2z = 8 \\ -4x + 2y + 2z = 5 \\ 2z = 10 \end{cases}$ $x = 2$; $y = 8$, $z = -5$
4. $\begin{cases} x + y + z = 15 \\ 2x - y + 3z = -5 \\ -x - y + z = -7 \end{cases}$ [Hint: Solve for z and get two equations to have x and y only.] $x = -2$; $y = 13$, $z = 4$

TEST PREP DOCTOR In **Exercise 36,** students who chose **A** may have confused the variables in the second equation. Remind students that the verb *is* or *was* often represents the equal sign in an equation. The second sentence in the exercise indicates that m will equal some expression that contains f, not vice versa.

Journal

Have students write about a price comparison they could make in their own lives and how they could compare the prices with a system of equations.

ALTERNATIVE ASSESSMENT

Have students write and solve a system of linear equations that requires using the Distributive Property in the solution. Then have students write a real-world application problem that can be represented by a system of linear equations in two variables.

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5-2 Lesson Quiz

Solve each system by substitution.

1. $\begin{cases} y = 2x \\ y = \frac{1}{2}x - 3 \end{cases}$ $(-2, -4)$

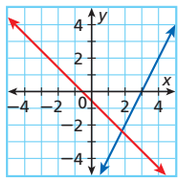
2. $\begin{cases} x = 6y - 11 \\ 3x - 2y = -1 \end{cases}$ $(1, 2)$

3. $\begin{cases} -3x + y = -1 \\ x - y = 4 \end{cases}$ $\left(-\frac{3}{2}, -\frac{11}{2}\right)$

4. Plumber A charges \$60 an hour. Plumber B charges \$40 to visit your home plus \$55 for each hour. For how many hours will the total cost for each plumber be the same? How much will that cost be? If a customer thinks they will need a plumber for 5 hours, which plumber should the customer hire? Explain.
   8 hours; \$480; plumber A; plumber A is cheaper for less than 8 hours.

Also available online

35. Estimation Use the graph to estimate the solution to $\begin{cases} 2x - y = 6 \\ x + y = -0.6 \end{cases}$. Round your answer to the nearest tenth.



Then solve the system by substitution.

Possible estimate: (1.75, −2.5); (1.8, −2.4)

TEST PREP

36. Elizabeth met 24 of her cousins at a family reunion. The number of male cousins m was 6 less than twice the number of female cousins f. Which system can be used to find the number of male cousins and female cousins?

Ⓐ $\begin{cases} m + f = 24 \\ f = 2m - 6 \end{cases}$ Ⓑ $\begin{cases} m + f = 24 \\ f = 2m \end{cases}$ Ⓒ $\begin{cases} m = 24 + f \\ m = f - 6 \end{cases}$ Ⓓ $\begin{cases} f = 24 - m \\ m = 2f - 6 \end{cases}$

37. Which problem is best represented by the system $\begin{cases} d = n + 5 \\ d + n = 12 \end{cases}$?

Ⓕ Roger has 12 coins in dimes and nickels. There are 5 more dimes than nickels.
Ⓖ Roger has 5 coins in dimes and nickels. There are 12 more dimes than nickels.
Ⓗ Roger has 12 coins in dimes and nickels. There are 5 more nickels than dimes.
Ⓙ Roger has 5 coins in dimes and nickels. There are 12 more nickels than dimes.

CHALLENGE AND EXTEND

38. A car dealership has 378 cars on its lot. The ratio of new cars to used cars is 5:4. Write and solve a system of equations to find the number of new and used cars on the lot.

$\begin{cases} n + u = 378 \\ 4n = 5u \end{cases}$;
210 new cars; 168 used cars

Solve each system by substitution.

39. $\begin{cases} 2r - 3s - t = 12 \\ s + 3t = 10 \\ t = 4 \end{cases}$ $r = 5; s = -2; t = 4$

40. $\begin{cases} x + y + z = 7 \\ y + z = 5 \\ 2y - 4z = -14 \end{cases}$ $x = 2; y = 1; z = 4$

41. $\begin{cases} a + 2b + c = 19 \\ -b + c = -5 \\ 3b + 2c = 15 \end{cases}$ $a = 9; b = 5; c = 0$

COMMON CORE

5-3 Solving Systems by Elimination

CC.9-12.A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. *Also* **CC.9-12.A.REI.6, CC.9-12.A.CED.3***

Objectives
Solve systems of linear equations in two variables by elimination.

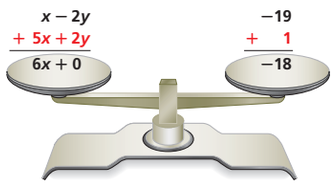
Compare and choose an appropriate method for solving systems of linear equations.

Why learn this?
You can solve a system of linear equations to determine how many flowers of each type you can buy to make a bouquet. (See Example 4.)

Photofusion Picture Library/Alamy

Another method for solving systems of equations is *elimination*. Like substitution, the goal of elimination is to get one equation that has only one variable.

Remember that an equation stays balanced if you add equal amounts to both sides. Consider the system $\begin{cases} x - 2y = -19 \\ 5x + 2y = 1 \end{cases}$. Since $5x + 2y = 1$, you can add $5x + 2y$ to one side of the first equation and 1 to the other side and the balance is maintained.



Since $-2y$ and $2y$ have **opposite coefficients**, you can eliminate the y by adding the two equations. The result is one equation that has only one variable: $6x = -18$.

When you use the elimination method to solve a system of linear equations, align all like terms in the equations. Then determine whether any like terms can be eliminated because they have opposite coefficients.

Know it! Note

Solving Systems of Equations by Elimination	
Step 1	Write the system so that like terms are aligned.
Step 2	Eliminate one of the variables and solve for the other variable.
Step 3	Substitute the value of the variable into one of the original equations and solve for the other variable.
Step 4	Write the answers from Steps 2 and 3 as an ordered pair, (x, y), and check.

Later in this lesson you will learn how to multiply one or more equations by a number in order to produce opposites that can be eliminated.

1 Introduce

EXPLORATION

Solving Systems by Elimination

In this Exploration, you will investigate another method for solving systems of equations.

1. Consider the following system of equations: $\begin{cases} 2x - y = 5 \\ x + y = 1 \end{cases}$.
   What can you say about the expressions on either side of the equal sign in the equation $x + y = 1$? The expressions are equal.
2. In general, what happens if you add equal quantities to both sides of an equation?
3. Explain why you can add $x + y$ to the left side of $2x - y = 5$ and add 1 to the right side of $2x - y = 5$.
4. Add $x + y$ to the left side of $2x - y = 5$ and add 1 to the right side of $2x - y = 5$. What is the resulting equation? $3x = 6$
5. Solve for x. $x = 2$
6. Use this value of x to find y. $y = -1$

THINK AND DISCUSS

7. **Explain** how you found the value of y once you knew the value of x. Substitute $x = 2$ in either of the equations and then solve for y.
8. **Describe** how you can check your solution to the system of equations. Use substitution to check that $(2, -1)$ is a solution of both equations.
2. The expressions on the two sides of the equation remain equal (or balanced).
3. Since the expressions are equal, you can add them to either side of the equation and preserve the balance.

Motivate

To introduce the word *elimination*, ask students what happens when the expressions below are simplified.

$5x + 2y + x - 2y$ $6x$; y is *eliminated.*

$2(x - y) - 2x + 7y$ $5y$; x is *eliminated.*

Tell students that in this lesson, they will eliminate variables to solve a system of equations.

Explorations and answers are provided in your online resources.

5-3 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Solve systems of linear equations in two variables by elimination.

Compare and choose an appropriate method for solving systems of linear equations.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Simplify each expression.

1. $3x + 2y - 5x - 2y$ $-2x$
2. $5(x - y) + 2x + 5y$ $7x$
3. $4y + 6x - 3(y + 2x)$ y
4. $2y - 4x - 2(4y - 2x)$ $-6y$

Write the least common multiple.

5. 3 and 6 6
6. 4 and 10 20
7. 6 and 8 24
8. 2 and 5 10

Also available online

Math Humor

Teacher: What's your solution of the system of equations?

Student: Nothing — I used elimination and got rid of the whole thing!

COMMON CORE

CC.9-12.A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
Also **CC.9-12.A.CED.3***

Learn It Online
State Resources Online

Teaching Tip **Communicating Math**
Some students may recognize the process of combining opposite terms as *cancellation* or *making zero pairs*. Use the word *eliminate* when describing the elimination method, but ask students to think of various names they have used to label this concept. This provides the students with a connection to previous learning.

ENGLISH LANGUAGE LEARNERS

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Additional Examples

Example 1

Solve $\begin{cases} 3x - 4y = 10 \\ x + 4y = -2 \end{cases}$ by elimination. Check your answer. $(2, -1)$

Example 2

Solve $\begin{cases} 2x + y = -5 \\ 2x - 5y = 13 \end{cases}$ by elimination. Check your answer. $(-1, -3)$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- Why is it helpful to write the system so that like terms are aligned?

EXAMPLE 2

- How do you decide what operation to use after you have aligned like terms?

Teaching Tip **Critical Thinking**
Challenge students to find two different ways to solve **Example 2** by substitution. One way is to solve each equation for $4y$ and then set the expressions equal to each other. Another is to divide the second equation by -2 and then isolate x.

EXAMPLE 1 Elimination Using Addition

Solve $\begin{cases} x - 2y = -19 \\ 5x + 2y = 1 \end{cases}$ by elimination.

Step 1
$x - 2y = -19$
$+ 5x + 2y = 1$ — *Notice that $-2y$ and $2y$ are opposites.*

Step 2
$6x + 0 = -18$ — *Add the equations to eliminate y.*
$6x = -18$ — *Simplify and solve for x.*
$\frac{6x}{6} = \frac{-18}{6}$ — *Divide both sides by 6.*
$x = -3$

Step 3
$x - 2y = -19$ — *Write one of the original equations.*
$-3 - 2y = -19$ — *Substitute -3 for x.*
$+3 \quad +3$ — *Add 3 to both sides.*
$-2y = -16$
$\frac{-2y}{-2} = \frac{-16}{-2}$ — *Divide both sides by -2.*
$y = 8$

Step 4 $(-3, 8)$ — *Write the solution as an ordered pair.*

Helpful Hint

Check your answer.

$x - 2y = -19$	
$-3 - 2(8)$	-19
$-3 - 16$	-19
-19	-19 ✓

$5x + 2y = 1$	
$5(-3) + 2(8)$	1
$-15 + 16$	1
1	1 ✓

CHECK IT OUT! **1.** Solve $\begin{cases} y + 3x = -2 \\ 2y - 3x = 14 \end{cases}$ by elimination. Check your answer. $(-2, 4)$

When two equations each contain the same term, you can subtract one equation from the other to solve the system. To subtract an equation, add the opposite of *each* term.

EXAMPLE 2 Elimination Using Subtraction

Solve $\begin{cases} 3x + 4y = 18 \\ -2x + 4y = 8 \end{cases}$ by elimination.

Step 1
$3x + 4y = 18$
$-(-2x + 4y = 8)$ — *Notice that both equations contain 4y.*
$3x + 4y = 18$
$+ 2x - 4y = -8$ — *Add the opposite of each term in the second equation.*

Step 2
$5x + 0 = 10$ — *Eliminate y.*
$5x = 10$ — *Simplify and solve for x.*
$x = 2$

Step 3
$-2x + 4y = 8$ — *Write one of the original equations.*
$-2(2) + 4y = 8$ — *Substitute 2 for x.*
$-4 + 4y = 8$
$+4 \quad +4$ — *Add 4 to both sides.*
$4y = 12$ — *Simplify and solve for y.*
$y = 3$

Step 4 $(2, 3)$ — *Write the solution as an ordered pair.*

Remember!

Remember to check by substituting your answer into both original equations.

2 Teach

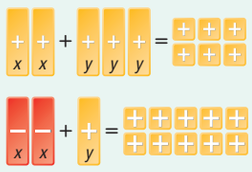
Guided Instruction

Review combining like terms before **Examples 1** and **2.** Review finding the least common multiple before **Example 3.** Be sure that students are comfortable with solving systems by addition or subtraction before showing them how to solve by multiplication. Discuss when each of the three methods of solving systems of linear equations is most convenient to use. Remind students that a solution is an ordered pair and that they should check their solutions.

Reaching All Learners

Through Concrete Manipulatives

Use transparency algebra tiles (MK) to model $2x + 3y = 6$ and $-2x + y = 10$. Label the x- and y-tiles.



Add the tiles vertically. Remove the zero pairs. This leaves $4y = 16$. Solve for y. Then find x through substitution. $(-3, 4)$

CHECK IT OUT! **2.** Solve $\begin{cases} 3x + 3y = 15 \\ -2x + 3y = -5 \end{cases}$ by elimination. Check your answer. **(4, 1)**

In some cases, you will first need to multiply one or both of the equations by a number so that one variable has opposite coefficients.

EXAMPLE 3 Elimination Using Multiplication First

Solve each system by elimination.

A $\begin{cases} 2x + y = 3 \\ -x + 3y = -12 \end{cases}$

Step 1

$2x + y = 3$

$+\ 2(-x + 3y = -12)$

Multiply each term in the second equation by 2 to get opposite x-coefficients.

$2x + y = 3$

$+\ (-2x + 6y = -24)$

Add the new equation to the first equation to eliminate x.

Step 2

$7y = -21$

$y = -3$ *Solve for y.*

Step 3

$2x + y = 3$ *Write one of the original equations.*

$2x + (-3) = 3$ *Substitute −3 for y.*

$+\ 3 \quad +3$ *Add 3 to both sides.*

$2x = 6$ *Solve for x.*

$x = 3$

Step 4 $(3, -3)$ *Write the solution as an ordered pair.*

B $\begin{cases} 7x - 12y = -22 \\ 5x - 8y = -14 \end{cases}$

Step 1

$2(7x - 12y = -22)$

$+\ (-3)(5x - 8y = -14)$

Multiply the first equation by 2 and the second equation by −3 to get opposite y-coefficients.

$14x - 24y = -44$

$+\ (-15x + 24y = 42)$

Add the new equations to eliminate y.

Step 2

$-x = -2$

$x = 2$ *Solve for x.*

Step 3

$7x - 12y = -22$ *Write one of the original equations.*

$7(2) - 12y = -22$ *Substitute 2 for x.*

$14 - 12y = -22$

$-14 \quad -14$ *Subtract 14 from both sides.*

$-12y = -36$ *Solve for y.*

$y = 3$

Step 4 $(2, 3)$ *Write the solution as an ordered pair.*

Helpful Hint

In Example 3A, you could have also multiplied the first equation by −3 to eliminate *y*.

CHECK IT OUT! **Solve each system by elimination. Check your answer.**

3a. $\begin{cases} 3x + 2y = 6 \\ -x + y = -2 \end{cases}$ **(2, 0)**

3b. $\begin{cases} 2x + 5y = 26 \\ -3x - 4y = -25 \end{cases}$ **(3, 4)**

COMMON ERROR ALERT

In problems like **Example 3,** students may forget to multiply both sides of the equation by a number when they create an opposite coefficient through multiplication. Remind them that the Multiplication Property of Equality requires both sides of the equation to be multiplied by the same number.

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Additional Examples

Example 3

Solve each system by elimination. Check your answer.

A. $\begin{cases} x + 2y = 11 \\ -3x + y = -5 \end{cases}$ **(3, 4)**

B. $\begin{cases} -5x + 2y = 32 \\ 2x + 3y = 10 \end{cases}$ **(−4, 6)**

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 3

- How do you choose the number to multiply each equation by?

Teaching Tip **Science Link** Systems of linear equations are commonly used in the sciences. A major field of study that uses systems of linear equations is digital signal processing (DSP). DSP has several applications, including weather forecasting, seismic (earth tremor) data processing, and image manipulation.

Teaching Tip **Number Sense** In **Example 4,** students can multiply both equations by 100 before solving. The new system will be equivalent to the original system, but it may be easier to solve because it will not contain decimals.

Teaching Tip **Visual** In **Example 4,** students may benefit from using a table to organize the information.

	Roses	Daises	Total
Number of Flowers	r	d	12
Cost of Flowers	\$2.50r	\$1.75d	\$24.75

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Additional Examples

Example 4

Paige has \$7.75 to buy 12 sheets of felt and card stock for her scrapbook. The felt costs \$0.50 per sheet, and the card stock costs \$0.75 per sheet. How many sheets of each can Paige buy?
5 sheets of felt and 7 sheets of card stock

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 4

- How do you know which numbers in the problem should be coefficients for the variables?

EXAMPLE 4 *Consumer Economics Application*

Sam spent \$24.75 to buy 12 flowers for his mother. The bouquet contained roses and daisies. How many of each type of flower did Sam buy?

Write a system. Use r for the number of roses and d for the number of daisies.

$2.50r + 1.75d = 24.75$ *The cost of roses and daisies totals \$24.75.*

$r + d = 12$ *The total number of roses and daisies is 12.*

Step 1

$2.50r + 1.75d = 24.75$

$+ (-2.50)(r + d = 12)$ *Multiply the second equation by −2.50 to get opposite r-coefficients.*

$2.50r + 1.75d = 24.75$

$+ (-2.50r - 2.50d = -30.00)$ *Add this equation to the first equation to eliminate r.*

Step 2

$-0.75d = -5.25$

$d = 7$ *Solve for d.*

Step 3

$r + d = 12$ *Write one of the original equations.*

$r + 7 = 12$ *Substitute 7 for d.*

$-7 \quad -7$ *Subtract 7 from both sides.*

$r = 5$

Step 4

$(5, 7)$ *Write the solution as an ordered pair.*

Sam can buy 5 roses and 7 daisies.

CHECK IT OUT! **4. What if...?** Sally spent \$14.85 to buy 13 flowers. She bought lilies, which cost \$1.25 each, and tulips, which cost \$0.90 each. How many of each flower did Sally buy? **9 lilies, 4 tulips**

All systems can be solved in more than one way. For some systems, some methods may be better than others.

Systems of Linear Equations

METHOD	USE WHEN...	EXAMPLE
Graphing	• Both equations are solved for y. • You want to estimate a solution.	$\begin{cases} y = 3x + 2 \\ y = -2x + 6 \end{cases}$
Substitution	• A variable in either equation has a coefficient of 1 or −1. • Both equations are solved for the same variable. • Either equation is solved for a variable.	$\begin{cases} x + 2y = 7 \\ x = 10 - 5y \end{cases}$ or $\begin{cases} x = 2y + 10 \\ x = 3y + 5 \end{cases}$
Elimination	• Both equations have the same variable with the same or opposite coefficients. • A variable term in one equation is a multiple of the corresponding variable term in the other equation.	$\begin{cases} 3x + 2y = 8 \\ 5x + 2y = 12 \end{cases}$ or $\begin{cases} 6x + 5y = 10 \\ 3x + 2y = 15 \end{cases}$

Victoria Smith/HMH

3 Close

Summarize

Ask students which method they would use to solve each system of equations and why. Possible answers:

$\begin{cases} x + 5y = 28 \\ -x + 2y = 7 \end{cases}$ Elimination using add.; the x-terms are opposites.

$\begin{cases} y = 6 \\ y = 4x - 2 \end{cases}$ Graphing: both in slope-int. form; substitution: both are solved for y.

$\begin{cases} -2x + 5y = -17 \\ 3x - 10y = 28 \end{cases}$ Elimination using mult.; the y-terms are multiples of 5.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

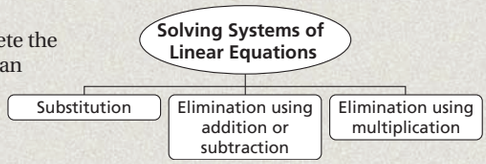
***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. Explain how multiplying the second equation in a system by −1 and eliminating by adding is the same as elimination by subtraction. Give an example of a system for which this applies.
2. Explain why it does not matter which variable you solve for first when solving a system by elimination.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of a system of equations that you could solve using the given method.

Know it! Note



5-3 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Solve each system by elimination. Check your answer.

SEE EXAMPLE 1

1. $\begin{cases} -x + y = 5 \\ x - 5y = -9 \end{cases}$ $(-4, 1)$
2. $\begin{cases} x + y = 12 \\ x - y = 2 \end{cases}$ $(7, 5)$
3. $\begin{cases} 2x + 5y = -24 \\ 3x - 5y = 14 \end{cases}$ $(-2, -4)$

SEE EXAMPLE 2

4. $\begin{cases} x - 10y = 60 \\ x + 14y = 12 \end{cases}$ $(40, -2)$
5. $\begin{cases} 5x + y = 0 \\ 5x + 2y = 30 \end{cases}$ $(-6, 30)$
6. $\begin{cases} -5x + 7y = 11 \\ -5x + 3y = 19 \end{cases}$ $(-5, -2)$

SEE EXAMPLE 3

7. $\begin{cases} 2x + 3y = 12 \\ 5x - y = 13 \end{cases}$ $(3, 2)$
8. $\begin{cases} -3x + 4y = 12 \\ 2x + y = -8 \end{cases}$ $(-4, 0)$
9. $\begin{cases} 2x + 4y = -4 \\ 3x + 5y = -3 \end{cases}$ $(4, -3)$

SEE EXAMPLE 4

10. **Consumer Economics** Each family in a neighborhood is contributing $20 worth of food to the neighborhood picnic. The Harlin family is bringing 12 packages of buns. The hamburger buns cost $2.00 per package. The hot-dog buns cost $1.50 per package. How many packages of each type of bun did they buy?

4 packages of hamburger buns, 8 packages of hot dog buns

PRACTICE AND PROBLEM SOLVING

Solve each system by elimination. Check your answer.

Independent Practice	
For Exercises	See Example
11–13	1
14–16	2
17–19	3
20	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

11. $\begin{cases} -x + y = -1 \\ 2x - y = 0 \end{cases}$ $(-1, -2)$
12. $\begin{cases} -2x + y = -20 \\ 2x + y = 48 \end{cases}$ $(17, 14)$
13. $\begin{cases} 3x - y = -2 \\ -2x + y = 3 \end{cases}$ $(1, 5)$
14. $\begin{cases} x - y = 4 \\ x - 2y = 10 \end{cases}$ $(-2, -6)$
15. $\begin{cases} x + 2y = 5 \\ 3x + 2y = 17 \end{cases}$ $\left(6, -\frac{1}{2}\right)$
16. $\begin{cases} 3x - 2y = -1 \\ 3x - 4y = 9 \end{cases}$ $\left(-\frac{11}{3}, -5\right)$
17. $\begin{cases} x - y = -3 \\ 5x + 3y = 1 \end{cases}$ $(-1, 2)$
18. $\begin{cases} 9x - 3y = 3 \\ 3x + 8y = -17 \end{cases}$ $\left(-\frac{1}{3}, -2\right)$
19. $\begin{cases} 5x + 2y = -1 \\ 3x + 7y = 11 \end{cases}$ $(-1, 2)$
20. **Multi-Step** Mrs. Gonzalez bought centerpieces to put on each table at a graduation party. She spent $31.50. There are 8 tables each requiring either a candle or vase. Candles cost $3 and vases cost $4.25. How many of each type did she buy?

20. 2 candles, 6 vases

Answers to *Think and Discuss*

1. Multiplying the second equation by −1 and adding is like adding the opposite, which is the same as subtracting; possible answer: $3x + y = 5$; $3x + 2y = 6$.
2. No matter which variable you solve for first, the answer will still be the same.
3. See Additional Answers.

5-3 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 11–16, 21, 23
Average 11–16, 21, 23
Advanced 11–16, 21, 23

If you finished Examples **1–4**
Basic 11–23, 31, 33–35
Average 11–21, 24–30 even, 31–39
Advanced 12–20 even, 21, 24–28 even, 31–39

Homework Quick Check
Quickly check key concepts.
Exercises: 12, 14, 18, 20

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 10, 20–21, 23, 30–31, 35, 39

Construct viable arguments and critique the reasoning of others. Exercises 22, 32

Look for and make use of structure. Exercises 24–29

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State Resources Online

Teaching Tip **Visual** Have students use a table like the one in **Exercise 23** to help them write systems of equations that represent real-world problems. To make a generic chart, they should label the left-hand rows *Situation 1* and *Situation 2*. The column headings should be *Item 1*, *Item 2*, and *Total*. The plus sign in the third column should be changed to +/−.

Multi-Step Test Prep **Exercise 31** involves writing and solving a system of equations to help calculate savings. This exercise prepares students for the Multi-Step Test Prep.

Answers

21. $\begin{cases} \ell - w = 2 \\ 2\ell + 2w = 40 \end{cases}$; length: 11 units; width: 9 units

24. $(40, -2)$; possible answer: elimination using subtraction; coefficients of x-values are the same.

25. $(3, 3)$; possible answer: elimination using multiplication; no variables have the same coefficient, so they cannot be eliminated by addition or subtraction.

26. $(3, -1)$; possible answer: elimination using subtraction; the y-terms have the same coefficients.

27. $\left(\frac{46}{7}, \frac{8}{7}\right)$; possible answer: I would use substitution because the second equation is already solved for a variable.

28–29. See Additional Answers.

21. Geometry The difference between the length and width of a rectangle is 2 units. The perimeter is 40 units. Write and solve a system of equations to determine the length and width of the rectangle. (*Hint:* The perimeter of a rectangle is $2\ell + 2w$.)

22. ERROR ANALYSIS Which is incorrect? Explain the error.

22. A; the student did not distribute the neg. sign to all values in the parentheses and so mistakenly added 3 to −3.

A

$$\begin{cases} x + y = -3 \\ 3x + y = 3 \end{cases} \qquad \begin{array}{r} x + y = -3 \\ -(3x + y = 3) \\ \hline -2x = 0 \\ x = 0 \end{array}$$

B

$$\begin{cases} x + y = -3 \\ 3x + y = 3 \end{cases} \qquad \begin{array}{r} x + y = -3 \\ -(3x + y = 3) \\ \hline -2x = -6 \\ x = 3 \end{array}$$

23. Chemistry A chemist has a bottle of a 1% acid solution and a bottle of a 5% acid solution. She wants to mix the two solutions to get 100 mL of a 4% acid solution. Follow the steps below to find how much of each solution she should use.

Math History

In 1247, Qin Jiushao wrote *Mathematical Treatise in Nine Sections*. Its contents included solving systems of equations and the Chinese Remainder Theorem.

	1% Solution	+	5% Solution	=	4% Solution
Amount of Solution (mL)	x	+	y	=	100
Amount of Acid (mL)	$0.01x$	+	$0.05y$	=	$0.04(100)$

a. Copy and complete the table.

b. Use the information in the table to write a system of equations. $\begin{cases} x + y = 100 \\ 0.01x + 0.05y = 4 \end{cases}$

c. Solve the system of equations to find how much she will use from each bottle to get 100 mL of a 4% acid solution. **25 mL of the 1% solution, 75 mL of the 5% solution**

Critical Thinking **Which method would you use to solve each system? Explain.**

24. $\begin{cases} \frac{1}{2}x - 5y = 30 \\ \frac{1}{2}x + 7y = 6 \end{cases}$

25. $\begin{cases} -x + 2y = 3 \\ 4x - 5y = -3 \end{cases}$

26. $\begin{cases} 3x - y = 10 \\ 2x - y = 7 \end{cases}$

27. $\begin{cases} 3y + x = 10 \\ x = 4y + 2 \end{cases}$

28. $\begin{cases} y = -4x \\ y = 2x + 3 \end{cases}$

29. $\begin{cases} 2x + 6y = 12 \\ 4x + 5y = 15 \end{cases}$

30. Business A local boys club sold 176 bags of mulch and made a total of $520. They did not sell any of the expensive cocoa mulch. Use the table to determine how many bags of each type of mulch they sold. **48 bags hardwood mulch; 128 bags pine bark mulch**

Mulch Prices ($)	
Cocoa	4.75
Hardwood	3.50
Pine Bark	2.75

31a. $\begin{cases} 3A + 2B = 16 \\ 2A + 3B = 14 \end{cases}$

Multi-Step Test Prep

31. a. The school store is running a promotion on school supplies. Different supplies are placed on two shelves. You can purchase 3 items from shelf A and 2 from shelf B for $16. Or you can purchase 2 items from shelf A and 3 from shelf B for $14. Write a system of equations that can be used to find the individual prices for the supplies on shelf A and on shelf B.

b. Solve the system of equations by elimination. $A = 4$; $B = 2$

c. If the supplies on shelf A are normally $6 each and the supplies on shelf B are normally $3 each, how much will you save on each package plan from part **a**? **Buying the first package will save $8; buying the second package will save $7.**

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PRACTICE A
PRACTICE C
PRACTICE B

Follow the steps to solve each system by elimination.

1. $\begin{cases} 2x - 3y = 14 \\ 2x + y = -10 \end{cases}$

Subtract the second equation:

$2x - 3y = 14$
$-(2x + y = -10)$
$-4y = 24$

Solve the resulting equation:

$y = -6$

Use your answer to find the value of x:

$x = -2$

Solution: $(-2, -6)$

2. $\begin{cases} 3x + y = 17 \\ 4x + 2y = 20 \end{cases}$

Multiply the first equation by −2. Then, add the equations:

$-6x - 2y = -34$
$+ 4x + 2y = 20$
$-2x = -14$

Solve the resulting equation:

$x = 7$

Use your answer to find the value of y:

$y = -4$

Solution: $(7, -4)$

Solve each system by elimination. Check your answer.

3. $\begin{cases} x + 3y = -7 \\ -x + 2y = -8 \end{cases}$ $(2, -3)$

4. $\begin{cases} 3x + y = -26 \\ 2x - y = -19 \end{cases}$ $(-9, 1)$

5. $\begin{cases} x + 3y = -14 \\ 2x - 4y = 32 \end{cases}$ $(4, -6)$

6. $\begin{cases} 4x - y = -5 \\ -2x + 3y = 10 \end{cases}$ $\left(-\frac{1}{2}, 3\right)$

7. $\begin{cases} y - 3x = 11 \\ 2y - x = 2 \end{cases}$ $(-4, -1)$

8. $\begin{cases} -10x + y = 0 \\ 5x + 3y = -7 \end{cases}$ $\left(-\frac{1}{5}, -2\right)$

Solve.

9. Brianna's family spent $134 on 2 adult tickets and 3 youth tickets at an amusement park. Max's family spent $146 on 3 adult tickets and 2 youth tickets. What is the price of a youth ticket? $22

10. Carl bought 19 apples of 2 different varieties to make a pie. The total cost of the apples was $5.10. Granny Smith apples cost $0.25 each and Gala apples cost $0.30 each. How many of each type of apple did Carl buy? 7 Gala apples; 12 Granny Smith apples

READING STRATEGIES

When solving systems of linear equations using elimination, you will sometimes need to multiply one or both equations by a factor in order to get the same coefficients for a variable. This process is very similar to getting a common denominator for fractions. Look at the example below.

$\begin{cases} 6x - 5y = 16 \\ 4x - 3y = 12 \end{cases}$ → To eliminate the x-terms, you need to get the same or opposite coefficients for x in both equations.

Add: $\frac{5}{6} + \frac{1}{4} = ?$ → Think about finding a common denominator for 4 and 6.

4: 4, 8, (12), 16, 20
6: 6, (12), 18, 24, 30
→ Find the least common multiple (LCM) of 4 and 6 by listing their multiples in order. The LCM is 12.

$\begin{cases} 2(6x - 5y = 16) \\ 3(4x - 3y = 12) \end{cases}$ → Determine what you have to multiple 4 and 6 by to get 12. Multiply each equation by the appropriate number.

$\begin{cases} 12x - 10y = 32 \\ 12x - 9y = 36 \end{cases}$ → Now either add or subtract the equations. In this case, you will subtract the equations.

1. Describe how you would get common y-coefficients (instead of x-) in the example above.
Multiply the first equation by 3 and the second equation by 5 to get common coefficients of −15.

2. Show how to get a set of common y-coefficients for the system $\begin{cases} 9x - 10y = 7 \\ 5x + 8y = 31 \end{cases}$.

$\begin{cases} 4(9x - 10y = 7) \\ 5(5x + 8y = 31) \end{cases} \Rightarrow \begin{cases} 36x - 40y = 28 \\ 25x + 40y = 155 \end{cases}$

Solve each system of equations by elimination.

3. $\begin{cases} 9x - 2y = 15 \\ 4x + 3y = -5 \end{cases}$ $(1, -3)$

4. $\begin{cases} 2x - 3y = 50 \\ 7x + 8y = -10 \end{cases}$ $(10, -10)$

RETEACH

Elimination can be used to solve a system of equations by adding terms vertically. This will cause one of the variables to be eliminated. It may be necessary to multiply one or both equations by some number to use this method.

I. Elimination may require no change to either equation.

$\begin{cases} 3x + y = 6 \\ 5x - y = 10 \end{cases}$ → Adding vertically will eliminate y. → $\begin{array}{r} 3x + y = 6 \\ 5x - y = 10 \\ \hline 8x + 0 = 16 \end{array}$

II. Elimination may require multiplying one equation by an appropriate number.

$\begin{cases} 2x + 5y = 9 \\ x - 3y = 10 \end{cases}$ → Multiply bottom equation by −2. → $\begin{array}{r} 2x + 5y = 9 \\ -2(x - 3y) = -2(10) \\ \hline 0 + 11y = -11 \end{array}$ $\begin{array}{r} 2x + 5y = 9 \\ -2x + 6y = -20 \\ \hline 0 + 11y = -11 \end{array}$

III. Elimination may require multiplying both equation by different numbers.

$\begin{cases} 5x + 3y = 2 \\ 4x + 2y = 10 \end{cases}$ → Multiply the top by −2 and the bottom by 3. → $\begin{array}{r} -2(5x + 3y = 2) \\ 3(4x + 2y = 10) \\ \hline 2x + 0 = 26 \end{array}$ $\begin{array}{r} -10x + -6y = -4 \\ 12x + 6y = 30 \\ \hline 2x + 0 = 26 \end{array}$

Solve each system by elimination.

1. $\begin{cases} 2x - y = 20 \\ 3x + 2y = -19 \end{cases}$ $(3, -14)$

2. $\begin{cases} 3x + 2y = 10 \\ 3x - 2y = 14 \end{cases}$ $(4, -1)$

3. $\begin{cases} x + y = 12 \\ 2x + y = 6 \end{cases}$ $(-6, 18)$

4. $\begin{cases} 3x - y = 2 \\ -8x + 2y = 4 \end{cases}$ $(-4, -14)$

32. Write About It Solve the system $\begin{cases} 3x + y = 1 \\ 2x + 4y = -6 \end{cases}$. Explain how you can check your solution algebraically and graphically.

TEST PREP

33. A math test has 25 problems. Some are worth 2 points, and some are worth 3 points. The test is worth 60 points total. Which system can be used to determine the number of 2-point problems and the number of 3-point problems on the test?

(A) $\begin{cases} x + y = 25 \\ 2x + 3y = 60 \end{cases}$ (B) $\begin{cases} x + y = 60 \\ 2x + 3y = 25 \end{cases}$ (C) $\begin{cases} x - y = 25 \\ 2x + 3y = 60 \end{cases}$ (D) $\begin{cases} x - y = 60 \\ 2x - 3y = 25 \end{cases}$

34. An electrician charges \$15 plus \$11 per hour. Another electrician charges \$10 plus \$15 per hour. For what amount of time will the cost be the same? What is that cost?

(F) 1 hour; \$25 (H) $1\frac{1}{2}$ hours; \$30

(G) $1\frac{1}{4}$ hours; \$28.75 (J) $1\frac{3}{4}$ hours; \$32.50

35. Short Response Three hundred fifty-eight tickets to the school basketball game on Friday were sold. Student tickets were \$1.50, and nonstudent tickets were \$3.25. The school made \$752.25.

a. Write a system of linear equations that could be used to determine how many student and how many nonstudent tickets were sold. Define the variables you use.

35a. Let s = the number of student tickets sold. Let n = the number of nonstudent tickets sold; $\begin{cases} s + n = 358 \\ 1.50s + 3.25n = 752.25. \end{cases}$

b. Solve the system you wrote in part **a.** How many student and how many nonstudent tickets were sold? $s = 235$; $n = 123$; 235 student tickets, 123 nonstudent tickets

CHALLENGE AND EXTEND

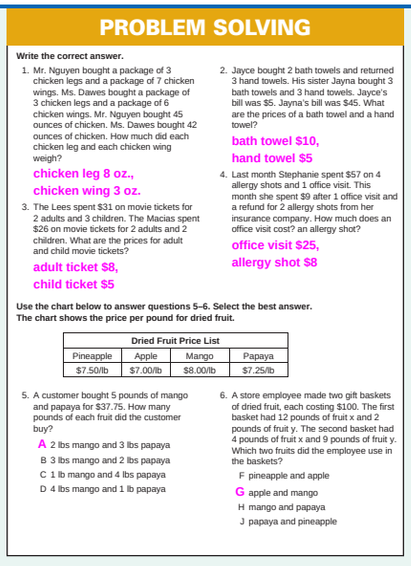
Solve each system by any method. $x = 4$; $y = -1$; $z = 10$

36. $\begin{cases} x + 16\frac{1}{2} = -\frac{3}{4}y \\ y = \frac{1}{2}x \end{cases}$ $x = -12$; $y = -6$

37. $\begin{cases} 2x + y + z = 17 \\ \frac{1}{2}z = 5 \\ x - y = 5 \end{cases}$

38. $\begin{cases} x - 2y - z = -1 \\ -x + 2y + 4z = -11 \\ 2x + y + z = 1 \end{cases}$ $x = 1$; $y = 3$; $z = -4$

39. The sum of the digits of a two-digit number is 5. If the number is multiplied by 3, the result is 42. Write and solve a system of equations to find the number. (*Hint:* One equation involves the digits in the number. The other equation involves the values of the digits.) $\begin{cases} x + y = 5 \\ 3(10x + y) = 42 \end{cases}$; $x = 1$; $y = 4$; the number is 14.



PROBLEM SOLVING

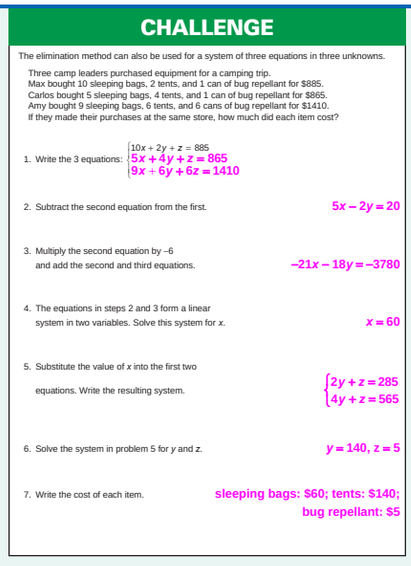
Write the correct answer.

1. Mr. Nguyen bought a package of 3 chicken legs and a package of 7 chicken wings. Ms. Dawes bought a package of 3 chicken legs and a package of 6 chicken wings. Mr. Nguyen bought 45 ounces of chicken. Ms. Dawes bought 42 ounces of chicken. How much did each chicken leg and each chicken wing weigh?
   chicken leg 8 oz., chicken wing 3 oz.
2. Jayce bought 2 bath towels and returned 3 hand towels. His sister Jayna bought 3 bath towels and 3 hand towels. Jayce's bill was \$5. Jayna's bill was \$45. What are the prices of a bath towel and a hand towel?
   bath towel \$10, hand towel \$5
3. The Lees spent \$31 on movie tickets for 2 adults and 3 children. The Macias spent \$26 on movie tickets for 2 adults and 2 children. What are the prices for adult and child movie tickets?
   adult ticket \$8, child ticket \$5
4. Last month Stephanie spent \$57 on 4 allergy shots and 1 office visit. This month she spent \$9 after 1 office visit and a refund for 2 allergy shots from her insurance company. How much does an office visit cost? an allergy shot?
   office visit \$25, allergy shot \$8

Use the chart below to answer questions 5–6. Select the best answer.
The chart shows the price per pound for dried fruit.

Dried Fruit Price List			
Pineapple	Apple	Mango	Papaya
\$7.50/lb	\$7.00/lb	\$8.00/lb	\$7.25/lb

5. A customer bought 5 pounds of mango and papaya for \$37.75. How many pounds of each fruit did the customer buy?
   A 2 lbs mango and 3 lbs papaya
   B 3 lbs mango and 2 lbs papaya
   C 1 lb mango and 4 lbs papaya
   D 4 lbs mango and 1 lb papaya
6. A store employee made two gift baskets of dried fruit, each costing \$100. The first basket had 12 pounds of fruit x and 2 pounds of fruit y. The second basket had 4 pounds of fruit x and 9 pounds of fruit y. Which two fruits did the employee use in the baskets?
   F pineapple and apple
   G apple and mango
   H mango and papaya
   J papaya and pineapple



CHALLENGE

The elimination method can also be used for a system of three equations in three unknowns.

Three camp leaders purchased equipment for a camping trip.
Max bought 10 sleeping bags, 2 tents, and 1 can of bug repellant for \$885.
Carlos bought 5 sleeping bags, 4 tents, and 1 can of bug repellant for \$865.
Amy bought 9 sleeping bags, 6 tents, and 6 cans of bug repellant for \$1410.
If they made their purchases at the same store, how much did each item cost?

1. Write the 3 equations: $\begin{cases} 10x + 2y + z = 885 \\ 5x + 4y + z = 865 \\ 9x + 6y + 6z = 1410 \end{cases}$
2. Subtract the second equation from the first. $5x - 2y = 20$
3. Multiply the second equation by –6 and add the second and third equations. $-21x - 18y = -3780$
4. The equations in steps 2 and 3 form a linear system in two variables. Solve this system for x. $x = 60$
5. Substitute the value of x into the first two equations. Write the resulting system. $\begin{cases} 2y + z = 285 \\ 4y + z = 565 \end{cases}$
6. Solve the system in problem 5 for y and z. $y = 140$, $z = 5$
7. Write the cost of each item. sleeping bags: \$60; tents: \$140; bug repellant: \$5

TEST PREP DOCTOR Students who chose **B** or **D** for **Exercise 33** may not have understood how to organize the information. Suggest that students use a table to organize the information into number of problems and number of points.

Answers

32. (1, –2); possible answer: to check the solution algebraically, substitute 1 for x and –2 for y in both equations. If both equations are true statements, the solution is correct. To check the solution graphically, graph both equations on the same coordinate plane and locate their intersection. If the intersection is (1, –2) then the solution is correct.

Journal

Have students write about which method for solving a system of equations they find the easiest.

ALTERNATIVE ASSESSMENT

Have students write a real-world situation that can be modeled by $\begin{cases} 4x + 8y = 100 \\ x + y = 15 \end{cases}$. Then have them solve the system and tell what the solution means in the context of their problem.

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5-3 Lesson Quiz

Solve each system by elimination.

1. $\begin{cases} 2x + y = 25 \\ 3y = 2x - 13 \end{cases}$ (11, 3)
2. $\begin{cases} -3x + 4y = -18 \\ x = -2y - 4 \end{cases}$ (2, –3)
3. $\begin{cases} -2x + 3y = -15 \\ 3x + 2y = -23 \end{cases}$ (–3, –7)
4. Harlan has \$44 to buy 7 pairs of socks. Athletic socks cost \$5 per pair. Dress socks cost \$8 per pair. How many pairs of each can Harlan buy?
   4 pairs of athletic socks and 3 pairs of dress socks

Also available online

Organizer

Pacing:
Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Apply solving systems of equations to number theory puzzles.

Online Edition

Countdown

Teach

Remember

Students review and apply number theory concepts to solve classic puzzle problems.

Reading Math
Recommend to students that they start their solution by creating a "dictionary" to identify the variables they are using for the system. If they get confused, they have a reference.

Learn It Online
State Resources Online

Solving Classic Problems

You can use systems of linear equations to solve some "classic" math problems that are common in textbooks and puzzle books.

Example 1

Yuri is twice as old as Zack. Four years from now, the sum of their ages will be 23. How old is Yuri?

Step 1 Write a system. Let y represent Yuri's age. Let z represent Zack's age.

Yuri is twice as old as Zack.

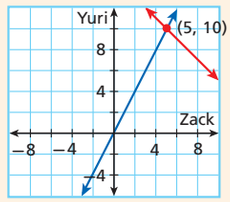
$$y = 2z$$

In 4 years, the sum of their ages will be 23.

$$(y + 4) + (z + 4) = 23$$
$$y + z + 8 = 23$$
$$y + z = 15$$

Step 2 Solve the equations for y.

$$\begin{cases} y = 2z \\ y + z = 15 \end{cases} \rightarrow \begin{cases} y = 2z \\ y = -z + 15 \end{cases}$$



Step 3 Graph $y = 2z$ and $y = -z + 15$.

The lines appear to intersect at $(5, 10)$.

The solution $(5, 10)$ means that Yuri is 10 years old.

Example 2

Mandy has 11 coins in dimes and quarters. The value of her coins is $2.15. How many dimes does she have?

Step 1 Write a system. Let d be the number of dimes. Let q be the number of quarters.

The total number of coins is 11.

$$q + d = 11$$

The value of the coins is $2.15.

$$0.25q + 0.10d = 2.15$$
$$100(0.25q + 0.10d = 2.15)$$
$$25q + 10d = 215$$
$$5q + 2d = 43$$

Step 2 Solve the first equation for d.

$$d = 11 - q$$

Step 3 Substitute $11 - q$ for d in the second equation.

$$5q + 2(11 - q) = 43$$
$$3q + 22 = 43 \quad \textit{Distribute 2 and then combine like terms.}$$
$$3q = 21$$
$$q = 7 \quad \textit{Solve for q.}$$

Step 4 Substitute 7 for q in one of the original equations.

$$q + d = 11$$
$$7 + d = 11$$
$$d = 4$$

The solution $(4, 7)$ means that there are 4 dimes and 7 quarters. Mandy has 4 dimes.

Example 3

When the digits of a two-digit number are reversed, the new number is 45 less than the original number. The sum of the digits is 7. What is the original number?

Step 1 Write expressions for the original number and the new number. Let a represent the tens digit. Let b represent the ones digit.

The original number: $10a + b$ The new number: $10b + a$

Step 2 Write a system.

The new number is 45 less than the original number. $10b + a = (10a + b) - 45$

The sum of the digits is 7. $a + b = 7$

$$9b - 9a = -45$$
$$b - a = -5 \rightarrow a - b = 5$$

Step 3 Add the equations to eliminate b. Solve for a.

$$\begin{array}{r} a - b = 5 \\ +\ (a + b = 7) \\ \hline 2a = 12 \end{array}$$
$$a = 6$$

Step 4 Substitute 6 for a in one of the original equations.

$$a + b = 7$$
$$6 + b = 7$$
$$b = 1$$

The solution is $(6, 1)$. This means that 6 is the tens digit and 1 is the ones digit. The original number is 61.

Check Check your solution using the original problem.
The sum of the digits is 7: $6 + 1 = 7$ ✓
When the digits are reversed, the new number is 45 less than the original number: $16 = 61 - 45$ ✓

Try This

1. The sum of the digits of a two-digit number is 17. When the digits are reversed, the new number is 9 more than the original number. What is the original number? **89**
2. Vic has 14 coins in nickels and quarters. The value of his coins is \$1.70. How many quarters does he have? **5 quarters**
3. Grace is 8 years older than her brother Sam. The sum of their ages is 24. How old is Grace? **16 yr old**

COMMON ERROR ALERT

Some students may forget to answer the question in the problem after solving the system of equations. Tell students that the solution to the system may not be the final answer, and remind them to make sure they answer the question in the problem.

Teaching Tip **Inclusion** It may be beneficial to have students practice writing systems of linear equations to model the scenarios in the *Try This* problems.

Close

Assess

Have students figure out the original number in **Example 3** if the new number is 27 more than the original. 25

Connecting Algebra to Number Theory **351**

5-4 Organizer

Pacing: Traditional 2 days
Block 1 day

Objectives: Solve special systems of linear equations in two variables.

Classify systems of linear equations and determine the number of solutions.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Solve each equation.

1. $2x + 3 = 2x + 4$ no solution

2. $2(x + 1) = 2x + 2$ infinitely many solutions

3. Solve $2y - 6x = 10$ for y. $y = 3x + 5$

Solve by using any method.

4. $\begin{cases} y = 3x + 2 \\ 2x + y = 7 \end{cases}$ $(1, 5)$

5. $\begin{cases} x - y = 8 \\ x + y = 4 \end{cases}$ $(6, -2)$

Also available online

Math Fact !!!

The solution set of an inconsistent system is the empty, or null, set, and can be symbolized with ∅ or { }.

COMMON CORE

CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.* *Also* **CC.9-12.A.CED.3***

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State Resources Online

COMMON CORE

5-4 Solving Special Systems

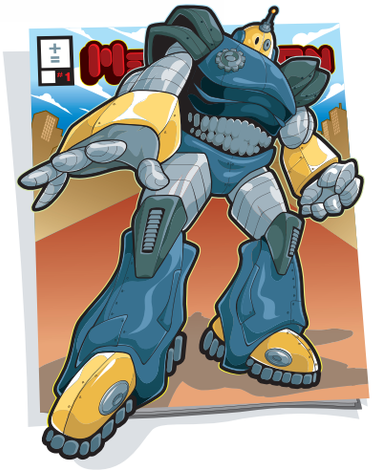
CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. *Also* **CC.9-12.A.CED.2***, **CC.9-12.A.CED.3***

Objectives
Solve special systems of linear equations in two variables.

Classify systems of linear equations and determine the number of solutions.

Vocabulary
consistent system
inconsistent system
independent system
dependent system

Why learn this?
Linear systems can be used to analyze business growth, such as comic book sales. (See Example 4.)



When two lines intersect at a point, there is exactly one solution to the system. A system with at least one solution is a **consistent system**.

When the two lines in a system do not intersect, they are parallel lines. There are no ordered pairs that satisfy both equations, so there is no solution. A system that has no solution is an **inconsistent system**.

EXAMPLE 1 **Systems with No Solution**

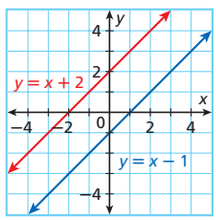
Show that $\begin{cases} y = x - 1 \\ -x + y = 2 \end{cases}$ has no solution.

Method 1 Compare slopes and y-intercepts.

$y = x - 1 \rightarrow y = 1x - 1$ *Write both equations in slope-intercept form.*
$-x + y = 2 \rightarrow y = 1x + 2$ *The lines are parallel because they have the same slope and different y-intercepts.*

This system has no solution.

Method 2 Graph the system.
The lines are parallel.



This system has no solution.

Method 3 Solve the system algebraically. Use the substitution method.

$-x + (x - 1) = 2$ *Substitute $x - 1$ for y in the second equation, and solve.*
$-1 = 2$ ✗ *False*

This system has no solution.

1. Possible answer: substitute $-2x + 5$ for y in the second equation.

$2x + (-2x + 5) = 1$
$5 = 1$ ✗

CHECK IT OUT! **1.** Show that $\begin{cases} y = -2x + 5 \\ 2x + y = 1 \end{cases}$ has no solution.

1 Introduce

EXPLORATION

Solving Special Systems

In this Exploration, you will use a graphing calculator to explore this system of equations: $\begin{cases} y = -\frac{1}{2}x + 3 \\ x + 2y = -4 \end{cases}$.

1. Enter the equations in your calculator as **Y1** and **Y2**. Remember that you must first write each equation in the form $y = \ldots$.
2. Press 2nd GRAPH (TABLE) to view a table of values for the equations.
3. Use the arrow keys to scroll up and down the table. Look for a solution of the system of equations by looking for a value of x for which the two y-values are equal. What do you notice?
4. Press GRAPH to view a graph of the functions. What do you notice about the graphs? The graph consists of two parallel lines.

THINK AND DISCUSS

5. **Describe** what happens when you try to solve this system of equations algebraically (by substitution or by elimination).
6. **Explain** why some systems of equations do not have a solution.

3. There is no solution because the y-values are never equal. They always differ by 5.
5. The process leads to a false statement, such as $6 = -4$.
6. When a system represents a pair of parallel lines, there is no solution.

Motivate

Review with students equations that have no solution, such as $x + 1 = x + 2$, and equations for which all real numbers are solutions, such as $2(x + 1) = 2x + 2$. Tell students that in this lesson, they will learn about systems of equations that either have no solution or infinitely many solutions.

Explorations and answers are provided in your online resources.

If two linear equations in a system have the same graph, the graphs are coincident lines, or the same line. There are infinitely many solutions of the system because every point on the line represents a solution of both equations.

EXAMPLE 2 Systems with Infinitely Many Solutions

Show that $\begin{cases} y = 2x + 1 \\ 2x - y + 1 = 0 \end{cases}$ has infinitely many solutions.

Method 1 Compare slopes and *y*-intercepts.

$y = 2x + 1 \rightarrow y = 2x + 1$
$2x - y + 1 = 0 \rightarrow y = 2x + 1$

Write both equations in slope-intercept form. The lines have the same slope and the same y-intercept.

If this system were graphed, the graphs would be the same line. There are infinitely many solutions.

Method 2 Solve the system algebraically. Use the elimination method.

$y = 2x + 1 \rightarrow -2x + y = 1$
$2x - y + 1 = 0 \rightarrow \underline{+2x - y = -1}$
$0 = 0$ ✓

Write equations to line up like terms.
Add the equations.
True. The equation is an identity.

There are infinitely many solutions.

Caution!

0 = 0 is a true statement. It does not mean the system has zero solutions or no solution.

CHECK IT OUT! **2.** Show that $\begin{cases} y = x - 3 \\ x - y - 3 = 0 \end{cases}$ has infinitely many solutions.

2. Possible answer: substitute $x - 3$ for y in the second equation.

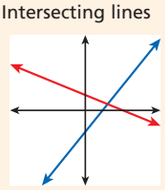
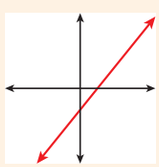
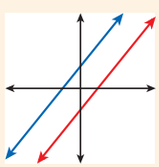
$x - (x - 3) - 3 = 0$
$3 - 3 = 0$
$0 = 0$ ✓

Consistent systems can either be independent or dependent.

- An **independent system** has exactly one solution. The graph of an independent system consists of two intersecting lines.
- A **dependent system** has infinitely many solutions. The graph of a dependent system consists of two coincident lines.

Know it! Note

Classification of Systems of Linear Equations

CLASSIFICATION	CONSISTENT AND INDEPENDENT	CONSISTENT AND DEPENDENT	INCONSISTENT
Number of Solutions	Exactly one	Infinitely many	None
Description	Different slopes	Same slope, same *y*-intercept	Same slope, different *y*-intercepts
Graph	Intersecting lines 	Coincident lines 	Parallel lines 

COMMON ERROR ALERT

Students might think that *infinitely many solutions* means that all ordered pairs are solutions. Remind them that the solutions are the infinite number of ordered pairs that make up the line.

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Additional Examples

Example 1

Show that $\begin{cases} y = x - 4 \\ -x + y = 3 \end{cases}$ has no solution.

Possible answer: Compare slopes and intercepts.

$y = x - 4$
$y = x + 3$

The lines have the same slope and diff. *y*-intercepts, so they are parallel.

Example 2

Show that $\begin{cases} y = 3x + 2 \\ 3x - y + 2 = 0 \end{cases}$ has infinitely many solutions.

Possible answer: Substitute $3x + 2$ for y in the second equation.

$3x - (3x + 2) + 2 = 0$
$0 = 0$ ✓

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- What is true about the graph of a system that is a false statement when it is solved algebraically?

EXAMPLE 2

- What is true about the graph of a system that is an identity when it is solved algebraically?

2 Teach

Guided Instruction

After introducing students to inconsistent systems in **Example 1** and consistent systems in **Example 2,** have students practice classifying systems. Be sure to make connections between the multiple representations of each type of system, as shown in the table.

Reaching All Learners

Through Auditory Cues

Have students work in pairs. One student describes either the graph, solution, or classification for a type of system. The other student gives the other two descriptions.

Graph	Solutions	Classification
Intersecting lines	Exactly one	Consistent, independent
Same line	Infinitely many	Consistent, dependent
Parallel lines	None	Inconsistent

Power Presentations with PowerPoint®

Additional Examples

Example 3

Classify each system. Give the number of solutions.

A. $\begin{cases} 3y = x + 3 \\ -\frac{1}{3}x + y = 1 \end{cases}$ consistent and dependent; infinitely many solutions

B. $\begin{cases} x + y = 5 \\ 4 + y = -x \end{cases}$ inconsistent; no solution

C. $\begin{cases} y = 4(x + 1) \\ y - 3 = x \end{cases}$ consistent and independent; one solution

Example 4

Jared and David both started a savings account in January. If the pattern of savings in the table continues, when will the amount in Jared's account equal the amount in David's account?

	Jan	Feb	Mar	Apr	May
Jared	25	30	35	40	45
David	40	45	50	55	60

The amount in Jared's account will never equal the amount in David's account.

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 3

- How are independent and dependent systems different?
- How are consistent and inconsistent systems different?
- Is it possible for a dependent system to be inconsistent?

EXAMPLE 4

- Why are the values in the first column of the table equal to the constants in the equations?
- How do you determine the rate from the table?
- What would the graph of the system look like?

Teaching Tip **Multiple Representations** Encourage students to check their solutions in **Example 3** by graphing.

EXAMPLE 3 Classifying Systems of Linear Equations

Classify each system. Give the number of solutions.

A $\begin{cases} 2y = x + 2 \\ -\frac{1}{2}x + y = 1 \end{cases}$

$2y = x + 2 \rightarrow y = \frac{1}{2}x + 1$ *Write both equations in slope-intercept form.*

$-\frac{1}{2}x + y = 1 \rightarrow y = \frac{1}{2}x + 1$ *The lines have the same slope and the same y-intercepts. They are the same.*

The system is consistent and dependent. It has infinitely many solutions.

B $\begin{cases} y = 2(x - 1) \\ y = x + 1 \end{cases}$

$y = 2(x - 1) \rightarrow y = 2x - 2$ *Write both equations in slope-intercept form.*

$y = x + 1 \rightarrow y = 1x + 1$ *The lines have different slopes. They intersect.*

The system is consistent and independent. It has one solution.

3a. consistent; dependent; infinitely many solutions

3b. consistent; independent; one solution

3c. inconsistent; no solution

CHECK IT OUT! **Classify each system. Give the number of solutions.**

3a. $\begin{cases} x + 2y = -4 \\ -2(y + 2) = x \end{cases}$ **3b.** $\begin{cases} y = -2(x - 1) \\ y = -x + 3 \end{cases}$ **3c.** $\begin{cases} 2x - 3y = 6 \\ y = \frac{2}{3}x \end{cases}$

EXAMPLE 4 *Business Application*

The sales manager at Comics Now is comparing its sales with the sales of its competitor, Dynamo Comics. If the sales patterns continue, will the sales for Comics Now ever equal the sales for Dynamo Comics? Explain.

Comic Books Sold per Year (thousands)	2005	2006	2007	2008
Comics Now	130	170	210	250
Dynamo Comics	180	220	260	300

Helpful Hint

The increase in sales is the difference between sales each year.

Use the table to write a system of linear equations. Let y represent the sales total and x represent the number of years since 2005.

	Sales total	equals	increase in sales per year	times	years	plus	beginning sales.
Comics Now	y	=	40	•	x	+	130
Dynamo Comics	y	=	40	•	x	+	180

$\begin{cases} y = 40x + 130 \\ y = 40x + 180 \end{cases}$

$y = 40x + 130$ *Both equations are in slope-intercept form.*

$y = 40x + 180$ *The lines have the same slope, but different y-intercepts.*

The graphs of the two equations are parallel lines, so there is no solution. If the patterns continue, sales for the two companies will never be equal.

4. Yes; the graphs of the two equations have different slopes so they intersect.

CHECK IT OUT! **4.** Matt has $100 in a checking account and deposits $20 per month. Ben has $80 in a checking account and deposits $30 per month. Will the accounts ever have the same balance? Explain.

3 Close

Summarize

Give students the system $\begin{cases} y = 3x + 8 \\ \square y = \square x + \square \end{cases}$.

Have them replace the boxes with numbers so that the system has

Possible answers given:

- one solution. 1, 2, 1
- infinitely many solutions. 2, 6, 16
- no solutions. 1, 3, 5

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

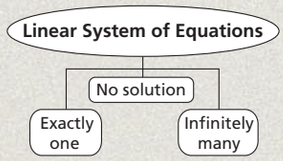
***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. What methods can be used to determine the number of solutions of a system of linear equations?
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write the word or words that describes a system with that number of solutions and sketch a graph.



Know it! Note

5-4 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** A _____?_____ system can be independent or dependent. (*consistent* or *inconsistent*) **consistent**

SEE EXAMPLE 1

Show that each system has no solution.

2. $\begin{cases} y = x + 1 \\ -x + y = 3 \end{cases}$
3. $\begin{cases} 3x + y = 6 \\ y = -3x + 2 \end{cases}$
4. $\begin{cases} -y = 4x + 1 \\ 4x + y = 2 \end{cases}$

SEE EXAMPLE 2

Show that each system has infinitely many solutions.

5. $\begin{cases} y = -x + 3 \\ x + y - 3 = 0 \end{cases}$
6. $\begin{cases} y = 2x - 4 \\ 2x - y - 4 = 0 \end{cases}$
7. $\begin{cases} -7x + y = -2 \\ 7x - y = 2 \end{cases}$

SEE EXAMPLE 3

Classify each system. Give the number of solutions.

8. $\begin{cases} y = 2x + 3 \\ -2y = 2x + 6 \end{cases}$
9. $\begin{cases} y = -3x - 1 \\ 3x + y = 1 \end{cases}$
10. $\begin{cases} 9y = 3x + 18 \\ \frac{1}{3}x - y = -2 \end{cases}$

SEE EXAMPLE 4

11. **Athletics** Micah walks on a treadmill at 4 miles per hour. He has walked 2 miles when Luke starts running at 6 miles per hour on the treadmill next to him. If their rates continue, will Luke's distance ever equal Micah's distance? Explain.
Yes; 1 hour after Luke begins, both boys will have covered 6 mi.

PRACTICE AND PROBLEM SOLVING

Show that each system has no solution.

12. $\begin{cases} y = 2x - 2 \\ -2x + y = 1 \end{cases}$
13. $\begin{cases} x + y = 3 \\ y = -x - 1 \end{cases}$
14. $\begin{cases} x + 2y = -4 \\ y = -\frac{1}{2}x - 4 \end{cases}$
15. $\begin{cases} -6 + y = 2x \\ y = 2x - 36 \end{cases}$

Show that each system has infinitely many solutions.

16. $\begin{cases} y = -2x + 3 \\ 2x + y - 3 = 0 \end{cases}$
17. $\begin{cases} y = x - 2 \\ x - y - 2 = 0 \end{cases}$
18. $\begin{cases} x + y = -4 \\ y = -x - 4 \end{cases}$
19. $\begin{cases} -9x - 3y = -18 \\ 3x + y = 6 \end{cases}$

Answers to *Think and Discuss*

1. solving algebraically by substitution or elimination, comparing slopes and *y*-intercepts, or graphing
2. See Additional Answers.

5-4 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 12–19, 26, 31
Average 12–19, 24–32 even
Advanced 12–19, 25–29 odd

If you finished Examples **1–4**
Basic 12–23, 26, 30–31, 33–34
Average 12–23, 24–32 even, 33–34
Advanced 12–23, 29–36

Homework Quick Check
Quickly check key concepts.
Exercises: 16, 18, 22, 23, 28

Answers

2. Possible answer: Substitute $x + 1$ for y in the second equation.
$-x + (x + 1) = 3$
$1 = 3$ ✗

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 11, 23–27, 30

Reason abstractly and quantitatively. Exercise 28

Construct viable arguments and critique the reasoning of others. Exercises 29, 31–32

Look for and make use of structure. Exercises 35–36

Learn It Online
State Resources Online

Answers

3. Possible answer: Substitute $-3x + 2$ for y in the first equation.
$3x + (-3x + 2) = 6$
$2 = 6$ ✗

4. Possible answer: Compare slopes and intercepts.
$-y = 4x + 1 \rightarrow y = -4x - 1$
$4x + y = 2 \rightarrow y = -4x + 2$
The lines have the same slope and different *y*-intercepts. Therefore the lines are parallel.

5. Possible answer: Substitute $-x + 3$ for y in the second equation.
$x + (-x + 3) - 3 = 0$
$0 = 0$ ✗

6. Possible answer: Substitute $2x - 4$ for y in the second equation.
$2x - (2x - 4) - 4 = 0$
$4 - 4 = 0$
$0 = 0$ ✗

7. Possible answer: Add the two equations.
$-7x + y = -2$
$+ \ 7x - y = 2$
$0 + 0 = 0$
$0 = 0$ ✗

8. cons., indep.; one sol.
9. inconsis.; no sol.
10. cons., dep.; inf. many solutions
12–19. See Additional Answers.

Teaching Tip **Critical Thinking** There are infinitely many answers to **Exercise 25.** Many students might choose $4 for a DVD and $1 for a video because that solution is easy to see. Encourage students to find decimal solutions, such as $3.50 and $2.

Teaching Tip **Visual** In dependent systems, such as in **Exercise 27,** have students substitute solutions of one equation into the other equation to show that the solutions of one equation are also solutions of the other.

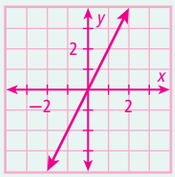
MULTI-STEP TEST PREP **Exercise 30** involves writing and solving a system of equations to compare cost. This exercise prepares students for the Multi-Step Test Prep.

Answers

24. No; the graphs of the two equations are parallel lines, so there is no solution.

25. There are inf. many answers for the cost of each video game and DVD. The system is consis. and dep.

28b.



coincident lines

c. The equations are the same.

d. The graphs will now be 2 parallel lines. The equations have the same slope but different *y*-intercepts.

30. See Additional Answers.

Independent Practice

For Exercises	See Example
12–15	1
16–19	2
20–22	3
23	4

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Classify each system. Give the number of solutions.

20. $\begin{cases} y = -x + 5 \\ x + y = 5 \end{cases}$ cons., dep.; inf. many solutions

21. $\begin{cases} y = -3x + 2 \\ y = 3x \end{cases}$ cons., indep.; one sol.

22. $\begin{cases} y - 1 = 2x \\ y = 2x - 1 \end{cases}$ inconsis.; no sol.

23. **Sports** Mandy is skating at 5 miles per hour. Nikki is skating at 6 miles per hour and started 1 mile behind Mandy. If their rates stay the same, will Mandy catch up with Nikki? Explain. Yes; the graphs of the two equations have different slopes, so they intersect.

24. **Multi-Step** Photocopier A can print 35 copies per minute. Photocopier B can print 35 copies per minute. Copier B is started and makes 10 copies. Copier A is then started. If the copiers continue, will the number of copies from machine A ever equal the number of copies from machine B? Explain.

25. **Entertainment** One week Trey rented 4 DVDs and 2 video games for $18. The next week he rented 2 DVDs and 1 video game for $9. Find the rental costs for each video game and DVD. Explain your answer.

26. Rosa bought 1 pound of cashews and 2 pounds of peanuts for $10. At the same store, Sabrina bought 2 pounds of cashews and 1 pound of peanuts for $11. Find the cost per pound for cashews and peanuts. cashews: $4/lb; peanuts: $3/lb

LINK Geology

Geodes are rounded, hollow rock formations. Most are partially or completely filled with layers of colored quartz crystals. The world's largest geode was discovered in Spain in 2000. It is 26 feet long and 5.6 feet high.

27. **Geology** Pam and Tommy collect geodes. Pam's parents gave her 2 geodes to start her collection, and she buys 4 every year. Tommy has 2 geodes that were given to him for his birthday. He buys 4 every year. If Pam and Tommy continue to buy the same amount of geodes per year, when will Tommy have as many geodes as Pam? Explain your answer. They will always have the same amount; both started with 2 and add 4 every year.

28. Use the data given in the tables.

x	3	4	5	6
y	6	8	10	12

x	12	13	14	15
y	24	26	28	30

a. Write an equation to describe the data in each table. $y = 2x$; $y = 2x$

b. Graph the system of equations from part **a.** Describe the graph.

c. How could you have predicted the graph by looking at the equations?

d. **What if...?** Each *y*-value in the second table increases by 1. How does this affect the graphs of the two equations? How can you tell how the graphs would be affected without actually graphing?

29. **Critical Thinking** Describe the graphs of two equations if the result of solving the system by substitution or elimination is the statement $1 = 3$. The graph will be 2 parallel lines.

MULTI-STEP TEST PREP

30. The Crusader pep club is selling team buttons that support the sports teams. They contacted Buttons, Etc. which charges $50 plus $1.10 per button, and Logos, which charges $40 plus $1.10 per button.

a. Write an equation for each company's cost.

b. Use the system from part **a** to find when the price for both companies is the same. Explain.

c. What part of the equation should the pep club negotiate to change so that the cost of Buttons, Etc. is the same as Logos? What part of the equation should change in order to get a better price?

(cl), David R. Frazier Photolibrary, Inc./Alamy; (bl), Victoria Smith/HMH

PRACTICE A

PRACTICE C

PRACTICE B

Solve each system of linear equations.

1. $\begin{cases} y = 2x - 3 \\ y - 2x = -3 \end{cases}$ infinitely many solutions

2. $\begin{cases} 3x + y = 4 \\ -3x = y - 7 \end{cases}$ no solution

3. $\begin{cases} y = -4x + 1 \\ 4x = -y - 6 \end{cases}$ no solution

4. $\begin{cases} y - x + 3 = 0 \\ x = y + 3 \end{cases}$ infinitely many solutions

Classify each system. Give the number of solutions.

5. $\begin{cases} y = 3(x - 1) \\ -y + 3x = 3 \end{cases}$ consistent, dependent; infinitely many solutions

6. $\begin{cases} y - 2x = 5 \\ x = y - 3 \end{cases}$ consistent, independent; one solution

7. Sabina and Lou are reading the same book. Sabina reads 12 pages a day. She had read 36 pages when Lou started the book, and Lou reads at a pace of 15 pages per day. If their reading rates continue, will Sabina and Lou ever be reading the same page on the same day? Explain. Yes. The graphs of the two equations have different slopes. They will intersect.

8. Brandon started jogging at 4 miles per hour. After he jogged 1 mile, his friend Anton started jogging along the same path at a pace of 4 miles per hour. If they continue to jog at the same rate, will Anton ever catch up with Brandon? Explain. No. The graphs of the two equations are parallel lines. They will never intersect.

READING STRATEGIES

The table below can help you answer questions about linear systems.

Classification	Number of Solutions	Similarities and Differences in $y = mx + b$	Description of Graphed Lines	Result of Solving with Algebra
Consistent, Independent	1	different slopes (m)	intersecting	values for x and y ex. $x = 3$, $y = -5$
Consistent, Dependent	infinitely many	same slope (m) same y-int. (b)	coincident	identity statement ex. $4 = 4$
Inconsistent	0	same slope (m) different y-int. (b)	parallel	false statement ex. $-2 = 3$

1. Mary Kate solved a system by elimination as shown below. Classify the system.

$\begin{cases} 2x + 3y = 5 \\ 2x + 3y = 7 \end{cases} \Rightarrow \begin{array}{r} 2x + 3y = 5 \\ -(2x + 3y = 7) \\ \hline 0 = -2 \end{array}$ inconsistent

2. Raul solved a system of equations by substitution. He ended up with an identity statement. How many solutions does his system have? infinitely many

3. How many solutions does a system have if, when graphed, the lines are the same line? infinitely many

4. Two equations in a system have the same slope. Which classification can NOT describe the system? consistent, independent

5. The graph of a system consists of two intersecting lines. How many solutions does the system have? one

Classify each system, give the number of solutions, and describe its graph.

6. $\begin{cases} y = -3x \\ y = -3x + 2 \end{cases}$ inconsistent; 0; parallel lines

7. $\begin{cases} y = -x + 4 \\ x + y = 4 \end{cases}$ consistent, dependent; infinitely many; coincident lines

8. $\begin{cases} y = 2x - 1 \\ x = y \end{cases}$ coincident lines 1; intersecting lines

RETEACH

When solving equations in one variable, it is possible to have one solution, no solutions, or infinitely many solutions. The same results can occur when graphing systems of equations.

Solve $\begin{cases} 4x + 2y = 2 \\ 2x + y = 4 \end{cases}$.

Multiplying the second equation by −2 will eliminate the *x*-terms.

$4x + 2y = 2$
$-2(2x + y = 4) \rightarrow -4x - 2y = -8$
$0 + 0 = -6$
$0 = -6$ ×

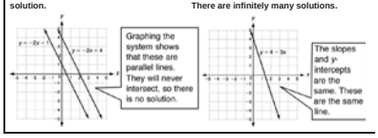
The equation is a contradiction. **There is no solution.**

Solve $\begin{cases} y = 4 - 3x \\ 3x + y = 4 \end{cases}$.

Because the first equation is solved for a variable, use substitution.

$3x + y = 4$
$3x + (4 - 3x) = 4$ *Substitute* $4 - 3x$ *for* y
$0 + 4 = 4$
$4 = 4$ ✓

The equation is true for all values of *x* and *y*. **There are infinitely many solutions.**



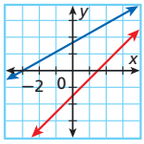
Solve each system of linear equations algebraically.

1. $\begin{cases} y = 3x \\ 2y = 6x \end{cases}$ infinitely many solutions

2. $\begin{cases} y = 2x + 5 \\ y - 2x = 1 \end{cases}$ no solution

3. $\begin{cases} 3x - 2y = 9 \\ -6x + 4y = 1 \end{cases}$ no solution

31. ///ERROR ANALYSIS/// Student A says there is no solution to the graphed system of equations. Student B says there is one solution. Which student is incorrect? Explain the error.



Student A; the lines are not parallel, so they will intersect.

32. Write About It Compare the graph of a system that is consistent and independent with the graph of a system that is consistent and dependent.

The graph of a system that is consistent and independent shows 2 lines that intersect at 1 point. The graph of a system that is consistent and dependent shows only 1 line.

TEST PREP

33. Which of the following classifications fit the following system?

$$\begin{cases} 2x - y = 3 \\ 6x - 3y = 9 \end{cases}$$

(A) Inconsistent and independent
(B) Consistent and independent
(C) Inconsistent and dependent
(D) Consistent and dependent

34. Which of the following would be enough information to classify a system of two linear equations?

(F) The graphs have the same slope.
(G) The *y*-intercepts are the same.
(H) The graphs have different slopes.
(J) The *y*-intercepts are different.

CHALLENGE AND EXTEND

35. What conditions are necessary for the system $\begin{cases} y = 2x + p \\ y = 2x + q \end{cases}$ to have infinitely many solutions? no solution?

$p = q$; $p \neq q$

36. Solve the systems in parts **a** and **b**. Use this information to make a conjecture about all solutions that exist for the system in part **c**.

a. $\begin{cases} 3x + 4y = 0 \\ 4x + 3y = 0 \end{cases}$ **(0, 0)**

b. $\begin{cases} 2x + 5y = 0 \\ 5x + 2y = 0 \end{cases}$ **(0, 0)**

c. $\begin{cases} ax + by = 0 \\ bx + ay = 0 \end{cases}$, for $a > 0$, $b > 0$, $a \neq b$ **(0, 0)**

TEST PREP DOCTOR In **Exercise 33,** students can eliminate choices **A** and **C** because inconsistent systems are not classified as independent or dependent.

For **Exercise 33,** suggest that students think of the graphs of each answer.

Journal

Have students explain how they can determine the number of solutions of a system of equations by comparing them by writing the equations in slope-intercept form and comparing *m* and *b*.

ALTERNATIVE ASSESSMENT

Have students work in pairs to make a poster that describes and classifies all three types of systems of equations. Have them include the result of solving each type algebraically.

Power Presentations with PowerPoint®

5-4 Lesson Quiz

Solve and classify each system.

1. $\begin{cases} y = 5x - 1 \\ 5x - y - 1 = 0 \end{cases}$ infinitely many solutions; consistent, dependent

2. $\begin{cases} y = 4 + x \\ -x + y = 1 \end{cases}$ no solution; inconsistent

3. $\begin{cases} y = 3(x + 1) \\ y = x - 2 \end{cases}$ $\left(-\frac{5}{2}, -\frac{9}{2}\right)$; consistent, independent

4. If the pattern in the table continues, when will sales for Hats Off equal sales for Tops?
never

Hat Sales (thousands)				
Year	2004	2005	2006	2007
Hats Off	6.2	8	9.8	11.6
Tops	3.5	5.3	7.1	8.9

Also available online

PROBLEM SOLVING

Write the correct answer.

1. Tyra and Charmian are training for a bike race. Tyra has logged 256 miles so far and rides 48 miles per week. Charmian has logged 125 miles so far and rides 48 miles per week. If these rates continue, will Tyra's distance ever equal Charmian's distance? Explain.
No; the graphs are parallel lines so there is no solution.

2. Metroplexpress and Local Express are courier companies. Metroplexpress charges $15 to pick up a package and $0.50 per mile. Local Express charges $10 to pick up a package and $0.55 per mile. Classify this system and find its solution, if any.
consistent and independent; 100 mi and $65

3. The Singhs start savings accounts for their twin boys. The accounts earn 5% annual interest. The initial deposit in each account is $200. Classify this system and find its solution, if any.
consistent and dependent; infinitely many solutions

4. Frank earns $8 per hour. Madison earns $7.50 per hour. Frank started working after Madison had already earned $300. If these rates continue, will Frank's earnings ever equal Madison's earnings? If so, when?
Yes; at 600 hours.

Select the best answer.

5. A studio apartment at The Oaks costs $400 per month plus a $350 deposit. A studio apartment at Crossroads costs $400 per month plus a $300 deposit. How many solutions does this system have?
A no solutions
B 1 solution
C 2 solutions
D an infinite number of solutions

6. Jane and Gary are both landscape designers. Jane charges $75 for a consultation plus $25 per hour. Gary charges $50 for a consultation plus $30 per hour. For how many hours will Jane's charges equal Gary's charges?
F never
G after 2 hours
H after 5 hours
J always

7. A tank filled with 75 liters of water loses 0.5 liter of water per hour. A tank filled with 50 liters of water loses 0.1 liter of water per hour. How would this system be classified?
A inconsistent
B dependent
C consistent and independent
D consistent and dependent

8. Simon is 3 years older than Renata. Five years ago, Renata was half as old as Simon is now. How old are Simon and Renata now?
F Simon is 13 and Renata is 10.
G Simon is 15 and Renata is 10.
H Simon is 16 and Renata is 8.
J Simon is 16 and Renata is 13.

CHALLENGE

The puzzle below contains 23 linear equations. You goal is to find paths through the puzzle such that each pair of equations along your path forms a special type of system.

You must begin in the **Start** square, and end in the **Finish** square. Moves can be horizontal, vertical, or diagonal between adjacent squares. Each individual path cannot cross itself, but your answers to 1 and 2 might cross.

Start → ↓	1 $y = x + 2$	2 $3x = 3y - 6$	3 $3x + y = -1$	4 $8y = 8x + 16$
5 $y = 2x + 3$	6 $y = 3x - 1$	7 $y - x = 2$	8 $\frac{1}{3}x + \frac{1}{3}y = 1$	9 $3x = y + 1$
10 $2y = 6x + 6$	11 $2x - y = -3$	12 $2x + 2y = 4$	13 $2y - 4 = 2x$	14 $x - y = -2$
15 $\frac{1}{2}y = \frac{3}{2}x + \frac{1}{2}$	16 $3y = 2x + 1$	17 $3x - y = 4$	18 $\frac{1}{6}y = \frac{1}{3}x + \frac{1}{2}$	19 $\frac{1}{2}y = \frac{1}{2}x + 1$
20 $10x + 15y = 5$	21 $-12x + 4y$ 5 8	22 $\frac{1}{2}y = \frac{1}{3}x + \frac{1}{6}$	23 $x = \frac{1}{3}y + 1$	↓ Finish

Find a path through the puzzle such that each pair of equations forms...

1. ...a consistent and dependent system.
Start –1 –2 –7 –13 –14 –19 – Finish

2. ...an inconsistent system.
Start –6 –10 –15 –21 –17 –23 – Finish

SECTION 5A

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

Online Edition

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Learn It Online
State Resources Online

CHAPTER 5

SECTION 5A

MULTI-STEP TEST PREP

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Systems of Equations

We've Got Spirit Some cheerleaders are going to sell spirit bracelets and foam fingers to raise money for traveling to away games.

Company	Design fee	Cost per item
Spirit for You	$35	$2.50
Go Team	$20	$3.00

1. Two companies, Spirit for You and Go Team, are interested in providing the foam fingers. The cheerleaders plan to sell 100 foam fingers. Based on this information, which company should they choose? Explain your reasoning.

2. The cheerleaders sold foam fingers for $5 and spirit bracelets for $4. They sold 40 more foam fingers than bracelets, and they earned $965. Write a system of equations to describe this situation.

2. $\begin{cases} f = b + 40 \\ 965 = 5f + 4b \end{cases}$

3. Solve this system using at least two different methods. Explain each method. $f = 125$, $b = 85$; check students' work.

4. Using the company you chose in Problem 1, how much profit did the cheerleaders make from the foam fingers alone? (*Hint:* profit = amount earned − expenses) $277.50

5. What is the maximum price the cheerleaders could pay for each spirit bracelet in order to make a total profit of $500? $1.38

1. Spirit for You; for 100 fingers Spirit charges $285 and Go Team charges $320.

(t), Victoria Smith/MHH; (cr), David Madison ©2005; (b), © Les Stone/CORBIS

INTERVENTION

Scaffolding Questions

1. What are the variables in this problem situation? The independent variable is the number of foam fingers. The dependent variable is the total cost.
2. How do you calculate total sales? For each item, multiply the number of items by the unit price, and then add the products.
3. What are the possible ways that you can solve a system of equations? Substitution, elimination, graphing, using a table
4. What are expenses? money spent to create products
5. Which method is best for this situation? Explain. Some students may prefer algebraic methods, while others will prefer using a graphing calculator.

Extension

If the cheerleaders needed to make a total profit of $600, would the maximum price they pay for the spirit bracelets be more or less than the answer in **Problem 5**? Explain. less; because they could not spend as much up front

READY TO GO ON?

Quiz for Lessons 5-1 Through 5-4

5-1 Solving Systems by Graphing

Tell whether the ordered pair is a solution of the given system.

1. $(-2, 1)$; $\begin{cases} y = -2x - 3 \\ y = x + 3 \end{cases}$ yes
2. $(9, 2)$; $\begin{cases} x - 4y = 1 \\ 2x - 3y = 3 \end{cases}$ no
3. $(3, -1)$; $\begin{cases} y = -\frac{1}{3}x \\ y + 2x = 5 \end{cases}$ yes

Solve each system by graphing.

4. $\begin{cases} y = x + 5 \\ y = \frac{1}{2}x + 4 \end{cases}$ $(-2, 3)$
5. $\begin{cases} y = -x - 2 \\ 2x - y = 2 \end{cases}$ $(0, -2)$
6. $\begin{cases} \frac{2}{3}x + y = -3 \\ 4x + y = 7 \end{cases}$ $(3, -5)$

7. **Banking** Christiana and Marlena opened their first savings accounts on the same day. Christiana opened her account with \$50 and plans to deposit \$10 every month. Marlena opened her account with \$30 and plans to deposit \$15 every month. After how many months will their two accounts have the same amount of money? What will that amount be? 4 mo; \$90

5-2 Solving Systems by Substitution

Solve each system by substitution.

8. $\begin{cases} y = -x + 5 \\ 2x + y = 11 \end{cases}$ $(6, -1)$
9. $\begin{cases} 4x - 3y = -1 \\ 3x - y = -2 \end{cases}$ $(-1, -1)$
10. $\begin{cases} y = -x \\ y = -2x - 5 \end{cases}$ $(-5, 5)$

5-3 Solving Systems by Elimination

Solve each system by elimination.

11. $\begin{cases} x + 3y = 15 \\ 2x - 3y = -6 \end{cases}$ $(3, 4)$
12. $\begin{cases} x + y = 2 \\ 2x + y = -1 \end{cases}$ $(-3, 5)$
13. $\begin{cases} -2x + 5y = -1 \\ 3x + 2y = 11 \end{cases}$ $(3, 1)$

14. It takes Akira 10 minutes to make a black and white drawing and 25 minutes for a color drawing. On Saturday he made a total of 9 drawings in 2 hours. Write and solve a system of equations to determine how many drawings of each type Akira made.

$10x + 25y = 120$; $x + y = 9$; 7 black and white, 2 color

5-4 Solving Special Systems

Solve each system of linear equations.

15. $\begin{cases} y = -2x - 6 \\ 2x + y = 5 \end{cases}$ no sol.
16. $\begin{cases} x + y = 2 \\ 2x + 2y = -6 \end{cases}$ no sol.
17. $\begin{cases} y = -2x + 4 \\ 2x + y = 4 \end{cases}$ inf. many solutions

Classify each system. Give the number of solutions.

18. $\begin{cases} 3x = -6y + 3 \\ 2y = -x + 1 \end{cases}$ cons., dep.; inf. many solutions
19. $\begin{cases} y = -4x + 2 \\ 4x + y = -2 \end{cases}$ incons.; no sol.
20. $\begin{cases} 4x - 3y = 8 \\ y = 4(x + 2) \end{cases}$ cons., indep.; one sol.

READY TO GO ON? SECTION 5A

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**
***Ready to Go On?* Online**

READY TO GO ON?

Diagnose and Prescribe

NO INTERVENE

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 5A		
	Worksheets	Online	
Lesson 5-1	5-1 Intervention	Activity 5-1	Diagnose and Prescribe Online
Lesson 5-2	5-2 Intervention	Activity 5-2	
Lesson 5-3	5-3 Intervention	Activity 5-3	
Lesson 5-4	5-4 Intervention	Activity 5-4	

YES ENRICH

***Ready to Go On? Enrichment*, Section 5A**
Worksheets
Online

SECTION 5B

Linear Inequalities

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 5-5 Solving Linear Inequalities • Graph and solve linear inequalities in two variables. ☑ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** graphing calculator, graph paper
Lesson 5-6 Solving Systems of Linear Inequalities • Graph and solve systems of linear inequalities in two variables. ☐ SAT-10 ☐ NAEP ☐ ACT ☑ SAT ☐ SAT Subject Tests		**Optional** colored pencils (MK), graphing calculator, transparency grids
Technology Lab Solve Systems of Linear Inequalities • Use a graphing calculator to view solutions of a system of linear inequalities. ☐ SAT-10 ☐ NAEP ☐ ACT ☑ SAT ☐ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** graphing calculator

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 5: 3, 5, 7, 9a, 9b, 9c, 11, 13, 15, 19a, 19b, 19c, 21, 23, 25, 29, 31, 33, 35, 37, 41, 43, 45, 47, 49

Lesson 6: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 45, 47, 49

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

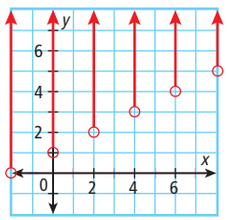
LINEAR INEQUALITIES

Lesson 5-5

A *linear inequality in two variables* is an inequality that may be written in the form $ax + by < c$, where a and b are not both 0, and where the symbols $\leq$, $>$, or $\geq$ may be used instead of $<$. A *solution* of a linear inequality in two variables is an ordered pair that makes the inequality true. For example, (4, 5) is a solution of the linear inequality $y > \frac{1}{2}x + 1$ because $5 > \frac{1}{2}(4) + 1$ is a true statement.

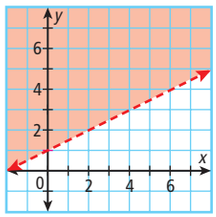
Students can begin to explore solutions of inequalities in two variables by fixing the value of one variable and then testing values of the other. For $y > \frac{1}{2}x + 1$, when $x = 4$ it is easy to see that any value of y greater than 3 is a solution. That is, the ordered pairs (4, 3.1), (4, 5), and (4, 17) are all solutions of $y > \frac{1}{2}x + 1$. More generally, the solution set when $x = 4$ can be represented by the ray described by $y > 3$. This can be shown on a coordinate plane as the part of the vertical line $x = 4$ where the y-values are greater than 3.

Similarly, the solution set when $x = 6$ can be represented by the ray described by $y > 4$. Doing this for other values of x results in a collection of rays.



If we imagine doing this for all values of x, we see that the solution set of $y > \frac{1}{2}x + 1$ can be represented by an infinite set of rays similar to those graphed above. These rays fill the part of the coordinate plane that lies above the line $y = \frac{1}{2}x + 1$. That is, the solution set can be represented by a half plane.

The shaded region shows the half plane that represents the solution set of $y > \frac{1}{2}x + 1$. All ordered pairs in this region are solutions of the inequality.



Note that the line $y = \frac{1}{2}x + 1$ is the boundary of the half plane. This boundary is shown with a dashed line because the points on the line are not solutions of the inequality. (For the inequality $y \geq \frac{1}{2}x + 1$, the points on the line *are* solutions and the boundary would be drawn as a solid line.)

Students should know how to check solutions to inequalities by selecting specific points from a graph. In the above graph, the point (4, 6) lies in the shaded region. Therefore, (4, 6) should be a solution of the inequality $y > \frac{1}{2}x + 1$. This is easily verified: $6 > \frac{1}{2}(4) + 1$ is a true statement. The point (5, 2) is *not* in the shaded region and so should *not* be a solution of the inequality. Again, this is easily verified by substituting 5 for x and 2 for y and noting that the resulting inequality is false.

SYSTEMS OF INEQUALITIES

Lesson 5-6

As discussed above, the solution set of a single inequality in two variables can be represented by a half plane. The solution set of a system of two inequalities in two variables can therefore be represented by the intersection of two half planes. Students should understand that this intersection may be an infinite region, a straight line, or the empty set. As before, specific points inside and outside the solution region should be checked in *both* inequalities to verify that the correct solution region has been identified.

5-5 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Graph and solve linear inequalities in two variables.

Algebra Lab
In Algebra Lab Activities

Online Edition
Tutorial Videos, Interactivity

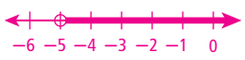
Countdown

Power Presentations with PowerPoint®

Warm Up

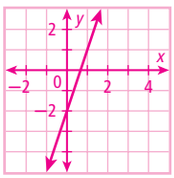
Graph each inequality.

1. $x > -5$



2. $y \leq 0$

3. Write $-6x + 2y = -4$ in slope-intercept form, and graph. $y = 3x - 2$



Also available online

Math Humor

Q: Why did the math student name her boundary line Hope?

A: Because it was dashed.

COMMON CORE

CC.9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half- planes.
CC.9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*

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COMMON CORE

5-5 Solving Linear Inequalities

CC.9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), …. *Also* **CC.9-12.A.CED.3***

Objective
Graph and solve linear inequalities in two variables.

Vocabulary
linear inequality
solution of a linear inequality

Who uses this?
Consumers can use linear inequalities to determine how much food they can buy for an event. (See Example 3.)

A **linear inequality** is similar to a linear equation, but the equal sign is replaced with an inequality symbol. A **solution of a linear inequality** is any ordered pair that makes the inequality true.

EXAMPLE 1 Identifying Solutions of Inequalities

Tell whether the ordered pair is a solution of the inequality.

A $(7, 3); y < x - 1$

y	$<$	$x - 1$
3		$7 - 1$
3	$<$	6 ✓

Substitute (7, 3) for (x, y).

$(7, 3)$ is a solution.

B $(4, 5); y > 3x + 2$

y	$>$	$3x + 2$
5		$3(4) + 2$
5		$12 + 2$
5	$>$	14 ✗

Substitute (4, 5) for (x, y).

$(4, 5)$ is not a solution.

CHECK IT OUT! **Tell whether the ordered pair is a solution of the inequality.**

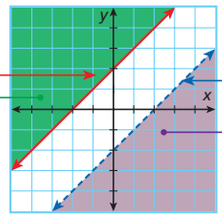
1a. $(4, 5); y < x + 1$ **no**

1b. $(1, 1); y > x - 7$ **yes**

A linear inequality describes a region of a coordinate plane called a *half-plane*. All points in the region are solutions of the linear inequality. The boundary line of the region is the graph of the related equation.

When the inequality is written as $y \leq$ or $y \geq$, the points on the boundary line are solutions of the inequality, and the line is **solid**.

When the inequality is written as $y <$ or $y >$, the points on the boundary line are not solutions of the inequality, and the line is **dashed**.



When the inequality is written as $y >$ or $y \geq$, the points **above** the boundary line are solutions of the inequality.

When the inequality is written as $y <$ or $y \leq$, the points **below** the boundary line are solutions of the inequality.

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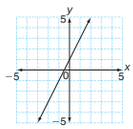
1 Introduce

EXPLORATION

Solving Linear Inequalities

You will need graph paper for this Exploration.
You will investigate how to solve linear inequalities.

1. Graph $y = 2x + 1$.
2. Choose three points above the line. Give the coordinates of each point and tell whether y is greater than, less than, or equal to $2x + 1$ at each point.
3. Now choose three points below the line. Give the coordinates of each point and tell whether y is greater than, less than, or equal to $2x + 1$ at each point.
4. Now choose three points on the line. Give the coordinates of each point and tell whether y is greater than, less than, or equal to $2x + 1$ at each point.



THINK AND DISCUSS

5. **Describe** the set of all points on the coordinate plane for which $y > 2x + 1$. the set of points above the line
6. **Describe** the set of all points on the coordinate plane for which $y \leq 2x + 1$. the set of points on or below the line

2. Possible answers: (1, 4), (−2, 2), (−3, −2); for all of the points, y is greater than $2x + 1$.
3. Possible answers: (0, 0), (3, 3), (−2, −4); for all of the points, y is less than $2x + 1$.
4. Possible answers: (−2, −3), (0, 1), (2, 5); for all of the points, y is equal to $2x + 1$.

Motivate

Pose some real-world situations that can be represented by linear inequalities in two variables. For example: The cost of 5 pencils and 2 pens must be at most \$4. $5x + 2y \leq 4$ Challenge students to find many possible solutions. Possible answers: (0.10, 1.50) = pencils \$0.10, pens \$1.50; (0.25, 1.00) = pencils \$0.25, pens \$1.00 Ask students if negative numbers are reasonable solutions for this problem. no

Explorations and answers are provided in your online resources.

Know it! Note

Graphing Linear Inequalities	
Step 1	Solve the inequality for y.
Step 2	Graph the boundary line. Use a solid line for $\leq$ or $\geq$. Use a dashed line for $<$ or $>$.
Step 3	Shade the half-plane above the line for $y >$ or $y \geq$. Shade the half-plane below the line for $y <$ or $y \leq$. Check your answer.

EXAMPLE 2 Graphing Linear Inequalities in Two Variables

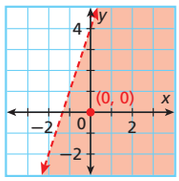
Graph the solutions of each linear inequality.

A $y < 3x + 4$

Step 1 The inequality is already solved for y.

Step 2 Graph the boundary line $y = 3x + 4$. Use a dashed line for $<$.

Step 3 The inequality is $<$, so shade below the line.



Check

y	$3x + 4$
0	$3(0) + 4$
0	$0 + 4$
0 $<$	4 ✓

Substitute (0, 0) for (x, y) because it is not on the boundary line.

The point (0, 0) satisfies the inequality, so the graph is shaded correctly.

Helpful Hint

The point (0, 0) is a good test point to use if it does not lie on the boundary line.

B $3x + 2y \geq 6$

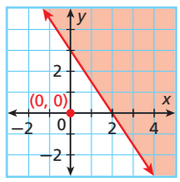
Step 1 Solve the inequality for y.

$3x + 2y \geq 6$

$-3x \quad -3x$

$2y \geq -3x + 6$

$y \geq -\frac{3}{2}x + 3$



Step 2 Graph the boundary line $y = -\frac{3}{2}x + 3$. Use a solid line for $\geq$.

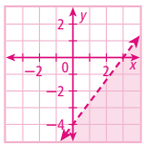
Step 3 The inequality is $\geq$, so shade above the line.

Check

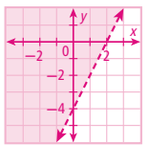
y	$\frac{3}{2}x + 3$
0	$\frac{3}{2}(0) + 3$
0	$0 + 3$
0 $\geq$	3 ✗

A false statement means that the half-plane containing (0, 0) should NOT be shaded. (0, 0) is not one of the solutions, so the graph is shaded correctly.

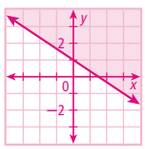
2a.



2b.



2c.



CHECK IT OUT! **Graph the solutions of each linear inequality.**

2a. $4x - 3y > 12$ **2b.** $2x - y - 4 > 0$ **2c.** $y \geq -\frac{2}{3}x + 1$

2 Teach

Guided Instruction

Explain to students that graphing an inequality begins with graphing an equation. The inequality symbol determines whether the points on the boundary line are solutions (solid line) or not solutions (dashed line). Ask students to think about what the shaded part will mean before they shade. Have students test several points in the area to be shaded to determine whether they make the inequality true.

Reaching All Learners

Through Home Connection

Have students search through local advertisements and find two different items (under $20) they would like to purchase. Tell students they have a maximum of $125 to purchase different combinations of these two items. Have students write a linear inequality to describe the situation. Possible answer: $13.99x + 17.95y \leq 125$ Then have them graph the solutions of their inequality and give at least two combinations of items they can purchase.

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Additional Examples

Example 1

Tell whether the ordered pair is a solution of the inequality.

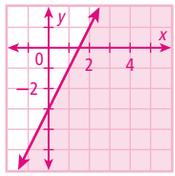
A. $(-2, 4)$; $y < 2x + 1$ no

B. $(3, 1)$; $y > x - 4$ yes

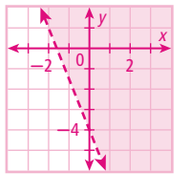
Example 2

Graph the solutions of each linear inequality.

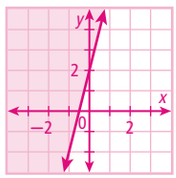
A. $y \leq 2x - 3$



B. $5x + 2y > -8$



C. $4x - y + 2 \leq 0$



Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- How do you know if an ordered pair is not a solution of a linear inequality?

EXAMPLE 2

- How can you check that you shaded the correct half-plane?
- Explain why the point $(0, 0)$ should not be used as a test point if it lies on the boundary line.

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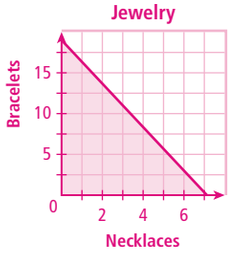
Additional Examples

Example 3

Ada has at most 285 beads to make jewelry. A necklace requires 40 beads, and a bracelet requires 15 beads. Write a linear inequality to describe the situation. Graph the solutions, describe reasonable solutions, and then give two combinations of necklaces and bracelets Ada could make.

$40x + 15y \leq 285$



Only whole-number solutions are reasonable. Possible answer: (2 necklaces, 8 bracelets), (5 necklaces, 3 bracelets)

Also available online

Questioning Strategies

EXAMPLE 3

- Can the reasonable solutions of a linear inequality include fractions or decimals? Why or why not?

Teaching Tip **Number Sense** In **Example 3,** discuss with students what numbers are reasonable solutions. In many real-world situations, reasonable solutions are often nonnegative, and sometimes only whole numbers are reasonable.

EXAMPLE 3 Consumer Economics Application

Sarah can spend at most $7.50 on vegetables for a party. Broccoli costs $1.25 per bunch and carrots cost $0.75 per package.

a. Write a linear inequality to describe the situation.

Let x represent the number of bunches of broccoli and let y represent the number of packages of carrots.

Write an inequality. Use $\leq$ for "at most."

Cost of broccoli	plus	cost of carrots	is at most	$7.50.
$1.25x$	+	$0.75y$	$\leq$	7.50

Solve the inequality for y.

$1.25x + 0.75y \leq 7.50$

$100(1.25x + 0.75y) \leq 100(7.50)$ *You can multiply both sides of the inequality by 100 to eliminate the decimals.*

$125x + 75y \leq 750$

$-125x \quad -125x$ *Subtraction Property of Inequality*

$75y \leq 750 - 125x$

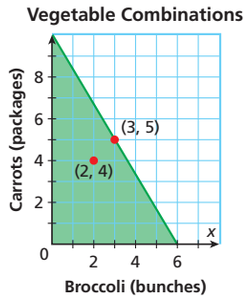
$\frac{75y}{75} \leq \frac{750 - 125x}{75}$ *Division Property of Inequality*

$y \leq 10 - \frac{5}{3}x$

b. Graph the solutions.

Step 1 Since Sarah cannot buy a negative amount of vegetables, the system is graphed only in Quadrant I. Graph the boundary line $y = -\frac{5}{3}x + 10$. Use a solid line for $\leq$.

Step 2 Shade below the line. Sarah must buy whole numbers of bunches or packages. All points on or below the line with whole-number coordinates represent combinations of broccoli and carrots that Sarah can buy.

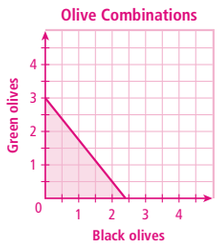


c. Give two combinations of vegetables that Sarah can buy.

Two different combinations that Sarah could buy for $7.50 or less are 2 bunches of broccoli and 4 packages of carrots, or 3 bunches of broccoli and 5 packages of carrots.

3a. $2.5b + 2g \leq 6$

3b.



3c. possible answer: (1 lb black, 1 lb green), (0.5 lb black, 2 lb green)

CHECK IT OUT!

3. Dirk is going to bring two types of olives to the Honor Society induction and can spend no more than $6. Green olives cost $2 per pound and black olives cost $2.50 per pound.

a. Write a linear inequality to describe the situation.

b. Graph the solutions.

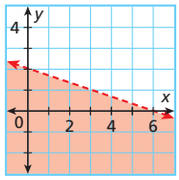
c. Give two combinations of olives that Dirk could buy.

Teaching Tip **Technology** Show students how to graph an inequality on a graphing calculator. Press Y= and enter the equation. Press ◀ to move the cursor to the left of Y_1. Press ENTER until the icon that looks like a region above or below a line appears. Press GRAPH. Tell students that the graphing calculator does not distinguish between solid and dashed boundary lines. They will need to decide for themselves if the line should be dashed or solid.

EXAMPLE 4 Writing an Inequality from a Graph

Write an inequality to represent each graph.

A



y-intercept: 2; slope: $-\frac{1}{3}$

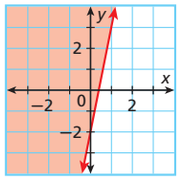
Write an equation in slope-intercept form.

$y = mx + b \longrightarrow y = -\frac{1}{3}x + 2$

The graph is shaded *below* a *dashed* boundary line.

Replace = with < to write the inequality $y < -\frac{1}{3}x + 2$.

B



y-intercept: −2; slope: 5

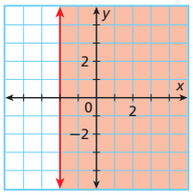
Write an equation in slope-intercept form.

$y = mx + b \longrightarrow y = 5x + (-2)$

The graph is shaded *above* a *solid* boundary line.

Replace = with ≥ to write the inequality $y \geq 5x - 2$.

C



y-intercept: none; slope: undefined

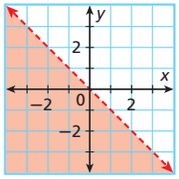
The graph is a vertical line at $x = -2$.

The graph is shaded on the *right* side of a *solid* boundary line.

Replace = with ≥ to write the inequality $x \geq -2$.

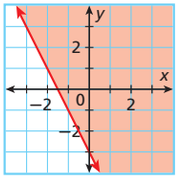
CHECK IT OUT! **Write an inequality to represent each graph.**

4a.



$y < -x$

4b.



$y \geq -2x - 3$

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Tell how graphing a linear inequality is the same as graphing a linear equation. Tell how it is different.
2. Explain how you would write a linear inequality from a graph.
3. **GET ORGANIZED** Copy and complete the graphic organizer.

Know it! Note

Inequality	$y < 5x + 2$	$y > 7x - 3$	$y \leq 9x + 1$	$y \geq -3x - 2$
Symbol	<			
Boundary Line	Dashed			
Shading	Below			

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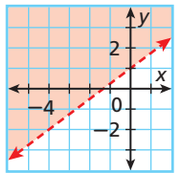
Additional Examples

Example 4

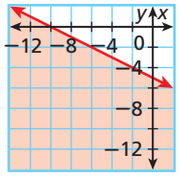
Write an inequality to represent each graph.

A.



$y > \frac{3}{4}x + 1$

B.



$y \leq -\frac{1}{2}x - 5$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 4

- How do you determine which inequality symbol to use?

3 Close

Summarize

For each inequality below, have students describe the boundary line and state which direction should be shaded.

1) $y < 2x + 1$ dashed; below
2) $y \geq -\frac{3}{5}x - 2$ solid; above
3) $2x - 3y \leq 6$ solid; above
4) $y > -3$ dashed; above
5) $x \leq 7$ solid; to the left

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Possible answer: In both cases you will always graph a line. They are different because the graph of a linear inequality can be a dashed or solid line, and the graph must be shaded on one side of the line.
2. To write a linear inequality from a graph, find the equation of the boundary line. Use ≥ or ≤ for a solid boundary line. Use > or < for a dashed boundary line. If the half-plane is shaded above the line, use > or ≥, and if it is shaded below, use < or ≤.
3. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
- **Basic** 12–18, 23–27
- **Average** 12–18, 23–27, 30–36 even
- **Advanced** 12–18, 23–27, 30–36 even

If you finished Examples **1–4**
- **Basic** 12–21, 22–36 even, 40–41, 43–45
- **Average** 12–21, 30–42 even, 43–48
- **Advanced** 12–21, 30–38 even, 39–40, 43-50

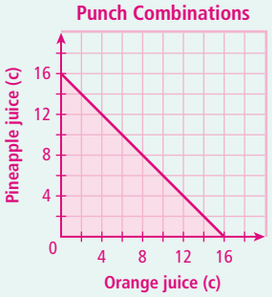
Homework Quick Check
Quickly check key concepts.
Exercises: 14, 18, 19, 20, 30, 40

Answers

5–8. See Additional Answers.

9a. $x + y \leq 16$

b.



c. Possible answer: (2 c orange, 2 c pineapple), (4 c orange, 10 c pineapple)

15–19. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 9, 19, 22, 27–28, 37, 40

Construct viable arguments and critique the reasoning of others. Exercises 38–39, 41–42

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5-5 Exercises

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Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** Can a *solution of a linear inequality* lie on a dashed boundary line? Explain. **No; a dashed line means that the ordered pairs are not solutions to the linear inequality.**

SEE EXAMPLE 1

Tell whether the ordered pair is a solution of the given inequality.

2. $(0, 3)$; $y \leq -x + 3$ **yes**
3. $(2, 0)$; $y > -2x - 2$ **yes**
4. $(-2, 1)$; $y < 2x + 4$ **no**

SEE EXAMPLE 2

Graph the solutions of each linear inequality.

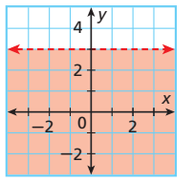
5. $y \leq -x$
6. $y > 3x + 1$
7. $-y < -x + 4$
8. $-y \geq x + 1$

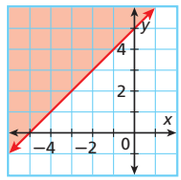
SEE EXAMPLE 3

9. **Multi-Step** Jack is making punch with orange juice and pineapple juice. He can make at most 16 cups of punch.
   - **a.** Write an inequality to describe the situation.
   - **b.** Graph the solutions.
   - **c.** Give two combinations of cups of orange juice and pineapple juice that Jack can use in his punch.

SEE EXAMPLE 4

Write an inequality to represent each graph.

10.  **$y < 3$**

11.  **$y \geq x + 5$**

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–14	1
15–18	2
19	3
20–21	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Tell whether the ordered pair is a solution of the given inequality.

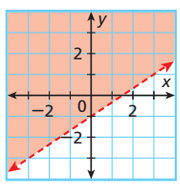
12. $(2, 3)$; $y \geq 2x + 3$ **no**
13. $(1, -1)$; $y < 3x - 3$ **yes**
14. $(0, 7)$; $y > 4x + 7$ **no**

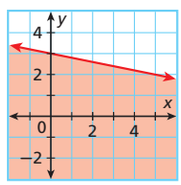
Graph the solutions of each linear inequality.

15. $y > -2x + 6$
16. $-y \geq 2x$
17. $x + y \leq 2$
18. $x - y \geq 0$

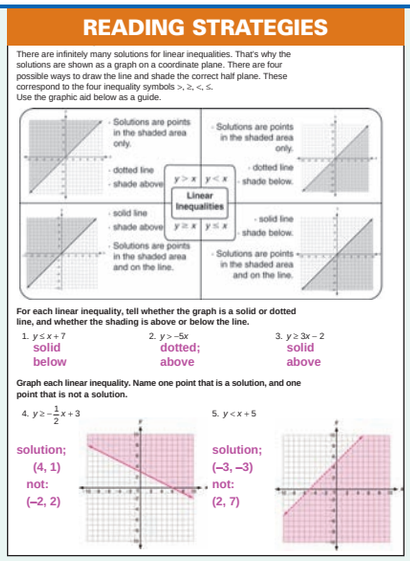
19. **Multi-Step** Beverly is serving hamburgers and hot dogs at her cookout. Hamburger meat costs \$3 per pound, and hot dogs cost \$2 per pound. She wants to spend no more than \$30.
   - **a.** Write an inequality to describe the situation.
   - **b.** Graph the solutions.
   - **c.** Give two combinations of pounds of hamburger and hot dogs that Beverly can buy.

Write an inequality to represent each graph.

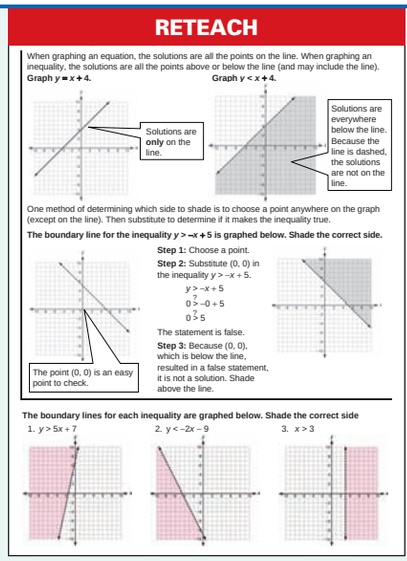
20.  **$y > \frac{2}{3}x - 1$**

21.  **$y \leq -\frac{1}{5}x + 3$**

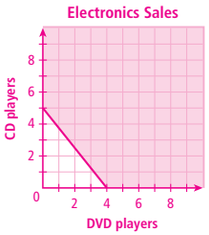
READING STRATEGIES



RETEACH



22b.



22. Business An electronics store makes $125 profit on every DVD player it sells and $100 on every CD player it sells. The store owner wants to make a profit of at least $500 a day selling DVD players and CD players. **a. $125x + 100y \geq 500$**

a. Write a linear inequality to determine the number of DVD players x and the number of CD players y that the owner needs to sell to meet his goal.

b. Graph the linear inequality. ***x* and *y* must be whole numbers.**

c. Describe the possible values of x. Describe the possible values of y.

d. List three combinations of DVD players and CD players that the owner could sell to meet his goal. **Possible answer: (5 DVD, 1 CD), (7 DVD, 6 CD), (9 DVD, 2 CD)**

Graph the solutions of each linear inequality.

23. $y \leq 2 - 3x$ **24.** $-y < 7 + x$ **25.** $2x - y \leq 4$ **26.** $3x - 2y > 6$

27. Geometry Marvin has 18 yards of fencing that he can use to put around a rectangular garden. **$2x + 2y \leq 18$**

a. Write an inequality to describe the possible lengths and widths of the garden.

b. Graph the inequality and list three possible solutions to the problem.

c. What are the dimensions of the largest *square* garden that can be fenced in with whole-number dimensions? **4 yd × 4 yd**

28. Hobbies Stephen wants to buy yellow tangs and clown fish for his saltwater aquarium. He wants to spend no more than $77 on fish. At the store, yellow tangs cost $15 each and clown fish cost $11 each. Write and graph a linear inequality to find the number of yellow tangs x and the number of clown fish y that Stephen could purchase. Name a solution of your inequality that is not reasonable for the situation. Explain.

Graph each inequality on a coordinate plane.

29. $y > 1$ **30.** $-2 < x$ **31.** $x \geq -3$ **32.** $y \leq 0$

33. $0 \geq x$ **34.** $-12 + y > 0$ **35.** $x + 7 < 7$ **36.** $-4 \geq x - y$

37. School At a high school football game, tickets at the gate cost $7 per adult and $4 per student. Write a linear inequality to determine the number of adult and student tickets that need to be sold so that the amount of money taken in at the gate is at least $280. Graph the inequality and list three possible solutions.

38. Critical Thinking Why must a region of a coordinate plane be shaded to show all solutions of a linear inequality? **Possible answer: It is not possible to list an infinite number of solutions, so shading is used to show all the solutions in the coordinate plane.**

39. Write About It Give a real-world situation that can be described by a linear inequality. Then graph the inequality and give two solutions.

Possible answer: Sammy volunteers at the hospital and the library. The total number of hours he volunteers is more than 5; $h + \ell > 5$; Check students' graphs and solutions.

MULTI-STEP TEST PREP

40. Gloria is making teddy bears. She is making boy and girl bears. She has enough stuffing to create 50 bears. Let x represent the number of girl bears and y represent the number of boy bears.

a. Write an inequality that shows the possible number of boy and girl bears Gloria can make. **$x + y \leq 50$**

b. Graph the inequality.

c. Give three possible solutions for the numbers of boy and girl bears that can be made. **Possible answer: (10, 20), (30, 10), (15, 15)**

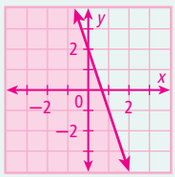
(l), Max Gibbs/photolibrary.com; (bl), Nathan Kaey/HMH; (cl), PhotoDisc Blue/Getty Images

Teaching Tip **Geometry** For **Exercise 27**, remind students that the formula for the perimeter of a rectangle is $P = 2\ell + 2w$ and the formula for the perimeter of a square is $P = 4s$.

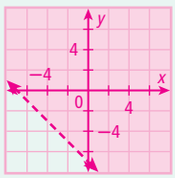
MULTI-STEP TEST PREP **Exercise 40** involves writing and graphing an inequality in two variables to determine possible solutions due to constraints on materials. This exercise prepares students for the Multi-Step Test Prep.

Answers

23.



24.



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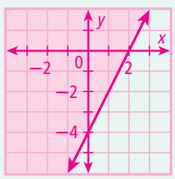
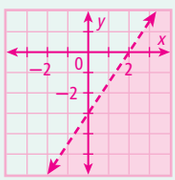


26.



27b, 28–37, 40b. See Additional Answers.

PRACTICE A

PRACTICE C

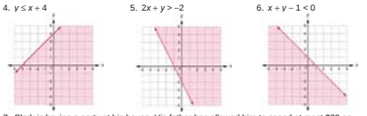
PRACTICE B

Tell whether the ordered pair is a solution of the given inequality.

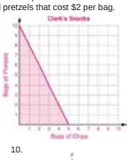
1. (1, 6); $y < x + 6$ — yes
2. (−3, −12); $y \geq 2x - 5$ — no
3. (5, −3); $y \leq -x + 2$ — yes

Graph the solutions of each linear inequality.

4. $y \leq x + 4$
5. $2x + y > -2$
6. $x + y - 1 < 0$



7. Clark is having a party at his house. His father has allowed him to spend at most $20 on snack food. He'd like to buy chips that cost $4 per bag, and pretzels that cost $2 per bag.
   a. Write an inequality to describe the situation. **Let x = chips, y = pretzels, $4x + 2y \leq 20$**
   b. Graph the solutions.
   c. Give two possible combinations of bags of chips and pretzels that Clark can buy. **Possible answer: 3 chips, 4 pretzels or 4 chips, 1 pretzel**



Write an inequality to represent each graph.

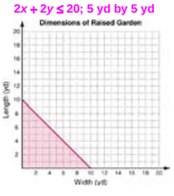
8. **$y \geq \frac{1}{2}x - 2$**
9. **$y < 3x + 1$**
10. **$y \leq -\frac{3}{2}x + 4$**

PROBLEM SOLVING

Write the correct answer.

1. Shania would like to give $5 gift cards and $4 teddy bears as party favors. Sixteen people have been invited to the party. Shania has $100 to spend on party favors. Write and graph an inequality to find the number of gift cards x and teddy bears y Shania could purchase. **$5x + 4y \leq 100$**

2. Hank has 20 yards of lumber that he can use to build a raised garden. Write and graph a linear inequality that describes the possible lengths and widths of the garden. If Hank wants the dimensions to be whole numbers only, what dimensions would produce the largest area? **$2x + 2y \leq 20$; 5 yd by 5 yd**



Select the best answer.

3. The royalties for the high school play are $250. Tickets to the play cost $5 for students and $8 for nonstudents. What linear inequality describes the number of student and nonstudent tickets that need to be sold so that the drama class can pay the royalties?
   A $5x + 8y \geq 250$ C $5xy + 8 < 250$
   B $5x + 8y > 250$ D $5xy + 8 \geq 250$

4. The inequality $x + y \leq 8$ describes the amounts of two juices Annette combines to make a smoothie. Which is a solution to the inequality?
   F (3, 6) H (7, 2)
   G (6, 1) J (0, 10)

5. A baker is making chocolate and lemon pound cakes. He can make at most 12 cakes at one time. Which inequality describes the situation?
   A $x + y > 12$ **C** $x + y \leq 12$
   B $x + y \geq 12$ D $x + y < 12$

6. Erasmus is the master gardener for a university. He wants to plant a mixture of purple and yellow pansies at the west entrance to the campus. From past experience, Erasmus knows that fewer than 350 pansies will fit in the planting area. Which inequality describes the situation?
   F $x + y \geq 350$ H $x + y \leq 350$
   G $x + y > 350$ **J** $x + y < 350$

CHALLENGE

To graph non-linear inequalities, follow the steps below.
Step 1: Solve the inequality for y.
Step 2: Write the related equation and generate ordered pairs.
Step 3: Plot enough points to see a pattern.
Step 4: Connect the points with a line or curve. Use a solid line for $\leq$ or $\geq$. Use a dashed line for $<$ or $>$.
Step 5: Shade above the line or curve for $y >$ or $y \geq$. Shade below for $y <$ or $y \leq$.

Generate ordered pairs and graph each inequality.

1. $y > (x + 3)^2 - 4$

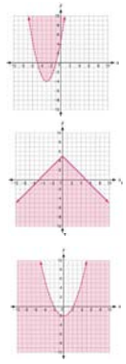
x	Think: $y = (x + 3)^2 - 4$	(x, y)
−6	$y = (-6 + 3)^2 - 4 = (-3)^2 - 4 = 9 - 4 = 5$	(−6, 5)
−5	$y = (-5 + 3)^2 - 4 = (-2)^2 - 4 = 4 - 4 = 0$	(−5, 0)
−4	$y = (-4 + 3)^2 - 4 = (-1)^2 - 4 = 1 - 4 = -3$	(−4, −3)
−3	$y = (-3 + 3)^2 - 4 = (-0)^2 - 4 = 0 - 4 = -4$	(−3, −4)
−2	$y = (-2 + 3)^2 - 4 = (1)^2 - 4 = 1 - 4 = -3$	(−2, −3)
−1	$y = (-1 + 3)^2 - 4 = (2)^2 - 4 = 4 - 4 = 0$	(−1, 0)

2. $|x| + y \leq 5$
   Solved for y: $y \leq -|x| + 5$

x	Think: $y = y = -\|x\| + 5$	(x, y)
−3	$y = -\|-3\| + 5 = -(3) + 5 = -3 + 5 = 2$	(−3, 2)
−2	$y = -\|-2\| + 5 = -(2) + 5 = -2 + 5 = 3$	(−2, 3)
−1	$y = -\|-1\| + 5 = -(1) + 5 = -1 + 5 = 4$	(−1, 4)
0	$y = -\|0\| + 5 = 0 + 5 = 5$	(0, 5)
1	$y = -\|1\| + 5 = -1 + 5 = 4$	(1, 4)
2	$y = -\|2\| + 5 = -2 + 5 = 3$	(2, 3)

3. $2x^2 - 4y > 8$
   Solved for y: $y < \frac{1}{2}x^2 - 2$

x	Think: $y = \frac{1}{2}x^2 - 2$	(x, y)
−4	$y = \frac{1}{2}(-4)^2 - 2 = \frac{1}{2}(16) - 2 = 8 - 2 = 6$	(−4, 6)
−2	$y = \frac{1}{2}(-2)^2 - 2 = \frac{1}{2}(4) - 2 - 2 - 2 = 0$	(−2, 0)
0	$y = \frac{1}{2}(0)^2 - 2 = \frac{1}{2}(0) - 2 = 0 - 2 = -2$	(0, −2)
1	$y = \frac{1}{2}(1)^2 - 2 = \frac{1}{2}(1) - 2 = \frac{1}{2} - 2 = -1\frac{1}{2}$	$\left(1, -1\frac{1}{2}\right)$
2	$y = \frac{1}{2}(2)^2 - 2 = \frac{1}{2}(4) - 2 = 2 - 2 = 0$	(2, 0)
4	$y = \frac{1}{2}(4)^2 - 2 = \frac{1}{2}(16) - 2 = 8 - 2 = 6$	(4, 6)



TEST PREP DOCTOR In **Exercise 44,** choices **G** and **J** can be eliminated because the boundary line is solid.

Journal

Have students describe the steps for graphing $y \leq x + 2$. Have them include how they would decide whether the boundary line is dashed or solid and how they would decide which half-plane to shade.

ALTERNATIVE ASSESSMENT

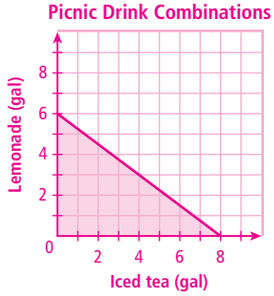
Have students write an inequality in two variables and graph the solutions. Have them explain what a solid boundary line and a dashed boundary line indicate about the solution set of a linear inequality.

Power Presentations with PowerPoint®

5-5 Lesson Quiz

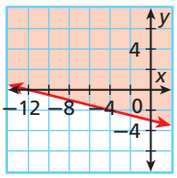
1. You can spend at most \$12.00 for drinks for a picnic. Iced tea costs \$1.50 per gallon, and lemonade costs \$2.00 per gallon. Write an inequality to describe the situation. Graph the solutions, describe reasonable solutions, and then give two possible combinations of drinks you could buy.

$1.50x + 2.00y \leq 12.00$



Only whole number solutions are reasonable. Possible answer: (2 gal iced tea, 3 gal lemonade), (4 gal iced tea, 1 gal lemonade)

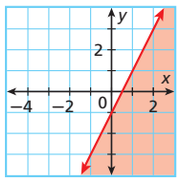
2. Write an inequality to represent the graph.



$y \geq -\frac{1}{4}x - 3$

Also available online

41. ///ERROR ANALYSIS/// Student A wrote $y < 2x - 1$ as the inequality represented by the graph. Student B wrote $y \leq 2x - 1$ as the inequality represented by the graph. Which student is incorrect? Explain the error.



Student A; the graph is a solid line.

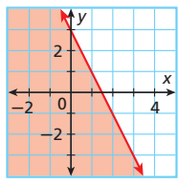
42. Write About It How do you decide to shade above or below a boundary line? What does this shading represent?

TEST PREP

43. Which point is a solution of the inequality $y > -x + 3$?

Ⓐ $(0, 3)$ Ⓑ $(1, 4)$ Ⓒ $(-1, 4)$ Ⓓ $(0, -3)$

44. Which inequality is represented by the graph at right?



Ⓕ $2x + y \geq 3$ Ⓗ $2x + y \leq 3$

Ⓖ $2x + y > 3$ Ⓙ $2x + y < 3$

45. Which of the following describes the graph of $3 \leq x$?

Ⓐ The boundary line is dashed, and the shading is to the right.

Ⓑ The boundary line is dashed, and the shading is to the left.

Ⓒ The boundary line is solid, and the shading is to the right.

Ⓓ The boundary line is solid, and the shading is to the left.

CHALLENGE AND EXTEND

Graph each inequality.

46. $0 \geq -6 - 2x - 5y$ **47.** $y > |x|$ **48.** $y \geq |x - 3|$

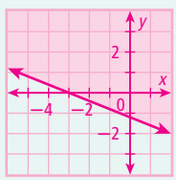
49. A linear inequality has the points $(0, 3)$ and $(-3, 1.5)$ as solutions on the boundary line. Also, the point $(1, 1)$ is not a solution. Write the linear inequality. $y \geq \frac{1}{2}x + 3$

50. Two linear inequalities are graphed on the same coordinate plane. The point $(0, 0)$ is a solution of both inequalities. The entire coordinate plane is shaded except for Quadrant I. What are the two inequalities? $x \leq 0$; $y \leq 0$

Answers

42. Possible answer: If the inequality is in slope-intercept form, then > or ≥ means you shade above the boundary line, and < or ≤ means you shade below the boundary line. The shading represents all the solutions of the inequality.

46.



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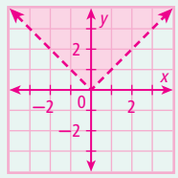
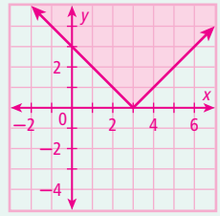


48.



COMMON CORE

5-6 Solving Systems of Linear Inequalities

CC.9-12.A.REI.12 Graph the … solution set to a system of linear inequalities in two variables as the intersection of the corresponding half- planes. *Also* **CC.9-12.A.CED.3***

Objective
Graph and solve systems of linear inequalities in two variables.

Vocabulary
system of linear inequalities
solutions of a system of linear inequalities

Who uses this?
The owner of a surf shop can use systems of linear inequalities to determine how many surfboards and wakeboards need to be sold to make a certain profit. (See Example 4.)

A **system of linear inequalities** is a set of two or more linear inequalities containing two or more variables. The **solutions of a system of linear inequalities** are all of the ordered pairs that satisfy all the linear inequalities in the system.

Remember!
An ordered pair must be a solution of all inequalities to be a solution of the system.

EXAMPLE 1 Identifying Solutions of Systems of Linear Inequalities

Tell whether the ordered pair is a solution of the given system.

A $(2, 1)$; $\begin{cases} y < -x + 4 \\ y \le x + 1 \end{cases}$

$(2, 1)$	$(2, 1)$
$y < -x + 4$	$y \le x + 1$
$1 \mid -2 + 4$	$1 \mid 2 + 1$
$1 < 2$ ✓	$1 \le 3$ ✓

$(2, 1)$ is a solution to the system because it satisfies both inequalities.

B $(2, 0)$; $\begin{cases} y \ge 2x \\ y < x + 1 \end{cases}$

$(2, 0)$	$(2, 0)$
$y \ge 2x$	$y < x + 1$
$0 \mid 2(2)$	$0 \mid 2 + 1$
$0 \ge 4$ ✗	$0 < 3$ ✓

$(2, 0)$ is not a solution to the system because it does not satisfy both inequalities.

CHECK IT OUT! Tell whether the ordered pair is a solution of the given system.

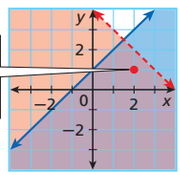
1a. $(0, 1)$; $\begin{cases} y < -3x + 2 \\ y \ge x - 1 \end{cases}$ yes

1b. $(0, 0)$; $\begin{cases} y > -x + 1 \\ y > x - 1 \end{cases}$ no

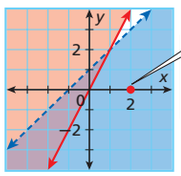
To show all the solutions of a system of linear inequalities, graph the solutions of each inequality. The solutions of the system are represented by the overlapping shaded regions. Below are graphs of Examples 1A and 1B.

Example 1A

(2, 1) is in the overlapping shaded regions, so it is a solution.



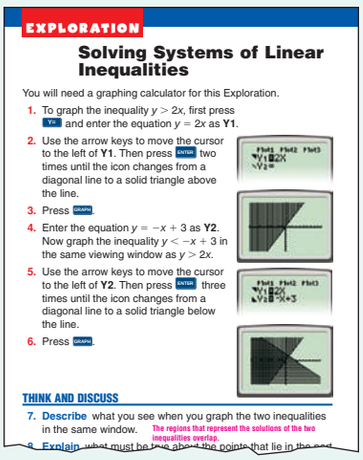
Example 1B



(2, 0) is not in the overlapping shaded regions, so it is not a solution.

© Justin Pumfrey/Taxi/Getty Images

1 Introduce



EXPLORATION

Solving Systems of Linear Inequalities

You will need a graphing calculator for this Exploration.

1. To graph the inequality $y > 2x$, first press Y= and enter the equation $y = 2x$ as **Y1**.
2. Use the arrow keys to move the cursor to the left of **Y1**. Press ENTER two times until the icon changes from a diagonal line to a solid triangle above the line.
3. Press GRAPH.
4. Enter the equation $y = -x + 3$ as **Y2**. Now graph the inequality $y < -x + 3$ in the same viewing window as $y > 2x$.
5. Use the arrow keys to move the cursor to the left of **Y2**. Then press ENTER three times until the icon changes from a diagonal line to a solid triangle below the line.
6. Press GRAPH.

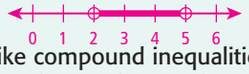
THINK AND DISCUSS

7. **Describe** what you see when you graph the two inequalities in the same window. The regions that represent the solutions of the two inequalities overlap.
8. **Explain** what must be true about the points that lie in the part

Motivate

Have students graph the solutions of the following compound inequality.

$x > 2$ AND $x < 5$



Tell students that, like compound inequalities, systems of inequalites have solutions that are represented by overlapping areas.

Explorations and answers are provided in your online resources.

5-6 Organizer

Pacing: Traditional 2 days
Block 1 day

Objective: Graph and solve systems of linear inequalities in two variables.

Online Edition
Tutorial Videos, Interactivity

Countdown

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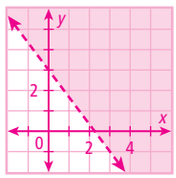
Warm Up

Solve each inequality for *y*.

1. $8x + y < 6$ $y < -8x + 6$
2. $3x - 2y > 10$ $y < \frac{3}{2}x - 5$
3. Graph the solutions of $4x + 3y > 9$.



Also available online

Math Humor

Q: Why did the math student wear two pairs of sunglasses?

A: He wanted to have overlapping shades.

COMMON CORE

CC.9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half- planes.
CC.9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*

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State Resources Online

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Additional Examples

Example 1

Tell whether the ordered pair is a solution of the given system.

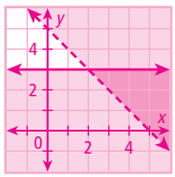
A. $(-1, -3)$; $\begin{cases} y \le -3x + 1 \\ y < 2x + 2 \end{cases}$ yes

B. $(-1, 5)$; $\begin{cases} y < -2x - 1 \\ y \ge x + 3 \end{cases}$ no

Example 2

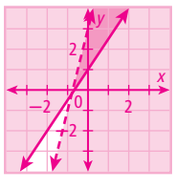
Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

A. $\begin{cases} y \le 3 \\ y > -x + 5 \end{cases}$



Possible answer: solutions: (8, 1), (6, 3); not solutions: (−4, 4), (2, 6)

B. $\begin{cases} -3x + 2y \ge 2 \\ y < 4x + 3 \end{cases}$

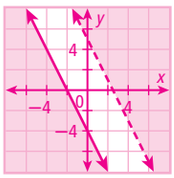


Possible answer: solutions: (2, 6), (1, 3); not solutions: (0, 0), (−4, 5)

Example 3

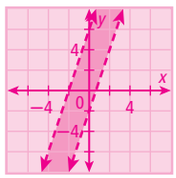
Graph each system of linear inequalities. Describe the solutions.

A. $\begin{cases} y \le -2x - 4 \\ y > -2x + 5 \end{cases}$



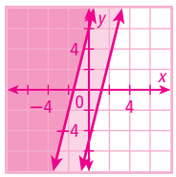
no solutions

B. $\begin{cases} y > 3x - 2 \\ y < 3x + 6 \end{cases}$



all pts. between the par. lines

C. $\begin{cases} y \ge 4x + 6 \\ y \ge 4x - 5 \end{cases}$



same as solutions of $y \ge 4x + 6$

Also available online

EXAMPLE 2 Solving a System of Linear Inequalities by Graphing

Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

$$\begin{cases} 8x + 4y \le 12 \\ y > \frac{1}{2}x - 2 \end{cases}$$

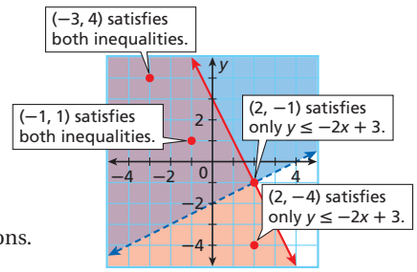
$8x + 4y \le 12$ *Solve the first inequality for y.*

$4y \le -8x + 12$

$y \le -2x + 3$

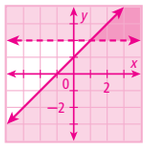
Graph the system.

$$\begin{cases} y \le -2x + 3 \\ y > \frac{1}{2}x - 2 \end{cases}$$



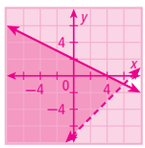
$(-1, 1)$ and $(-3, 4)$ are solutions.
$(2, -1)$ and $(2, -4)$ are not solutions.

2a.



Possible answer: solutions: (3, 3), (4, 4); not solutions: (−3, 1), (−1, −4)

2b.



Possible answer: solutions: (0, 0), (3, −2); not solutions: (4, 4), (1, −6)

CHECK IT OUT! **Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.**

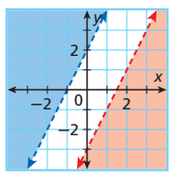
2a. $\begin{cases} y \le x + 1 \\ y > 2 \end{cases}$ **2b.** $\begin{cases} y > x - 7 \\ 3x + 6y \le 12 \end{cases}$

Previously, you saw that in systems of linear equations, if the lines are parallel, there are no solutions. With systems of linear inequalities, that is not always true.

EXAMPLE 3 Graphing Systems with Parallel Boundary Lines

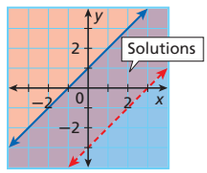
Graph each system of linear inequalities. Describe the solutions.

A $\begin{cases} y < 2x - 3 \\ y > 2x + 2 \end{cases}$



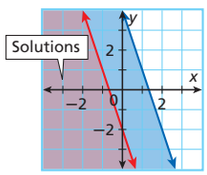
This system has no solution.

B $\begin{cases} y > x - 3 \\ y \le x + 1 \end{cases}$



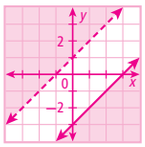
The solutions are all points between the parallel lines and on the solid line.

C $\begin{cases} y \le -3x - 2 \\ y \le -3x + 4 \end{cases}$

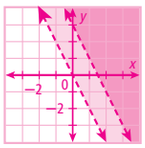


The solutions are the same as the solutions of $y \le -3x - 2$.

3a. no solutions



3b. all pts. between and on the parallel lines



2 Teach

Guided Instruction

Tell students that graphing a system of inequalities is just graphing two or more inequalities on the same coordinate plane. The solution is the overlapping region. Encourage students to always use a test point to check that they shaded the correct regions.

Teaching Tip **Visual** Have students shade graphs of inequalities using two different colored pencils (MK). This will make the solution area easier to see.

Reaching All Learners

Through Kinesthetic Experience

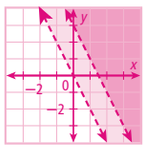
Have two students each graph a linear inequality in two variables on different transparencies. Then combine the two graphs by overlapping them, and have students write two ordered pairs that are solutions and two that are not. Repeat with another pair of students until all have taken a turn.

CHECK IT OUT! Graph each system of linear inequalities. Describe the solutions.

3a. $\begin{cases} y > x + 1 \\ y \le x - 3 \end{cases}$ **3b.** $\begin{cases} y \ge 4x - 2 \\ y \le 4x + 2 \end{cases}$ **3c.** $\begin{cases} y > -2x + 3 \\ y > -2x \end{cases}$

EXAMPLE 4 Business Application

3c. same as solutions of $y > -2x + 3$



A surf shop makes the profits given in the table. The shop owner sells at least 10 surfboards and at least 20 wakeboards per month. He wants to earn at least $2000 a month. Show and describe all possible combinations of surfboards and wakeboards that the store owner needs to sell to meet his goals. List two possible combinations.

Profit per Board Sold ($)	
Surfboard	150
Wakeboard	100

Step 1 Write a system of inequalities.

Let x represent the number of surfboards and y represent the number of wakeboards.

$x \ge 10$ *He sells at least 10 surfboards.*

$y \ge 20$ *He sells at least 20 wakeboards.*

$150x + 100y \ge 2000$ *He wants to earn a total of at least $2000.*

Step 2 Graph the system.

The graph should be in only the first quadrant because sales are not negative.

Step 3 Describe all possible combinations.

To meet the sales goals, the shop could sell any combination represented by an ordered pair of whole numbers in the solution region. Answers must be whole numbers because the shop cannot sell part of a surfboard or wakeboard.

Step 4 List two possible combinations.

Two possible combinations are:

15 surfboards and 25 wakeboards

25 surfboards and 20 wakeboards

Caution!

An ordered pair solution of the system need not have whole numbers, but answers to many application problems may be restricted to whole numbers.

4.

Possible answer: (3 lb pepper jack, 2 lb cheddar), (2.5 lb pepper jack, 4 lb cheddar)

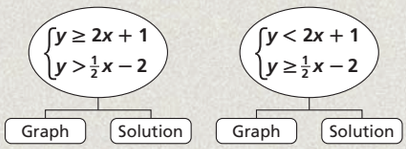
CHECK IT OUT! **4.** At her party, Alice is serving pepper jack cheese and cheddar cheese. She wants to have at least 2 pounds of each. Alice wants to spend at most $20 on cheese. Show and describe all possible combinations of the two cheeses Alice could buy. List two possible combinations.

Price per Pound ($)	
Pepper Jack	4
Cheddar	2

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. How would you write a system of linear inequalities from a graph?
2. **GET ORGANIZED** Copy and complete each part of the graphic organizer. In each box, draw a graph and list one solution.



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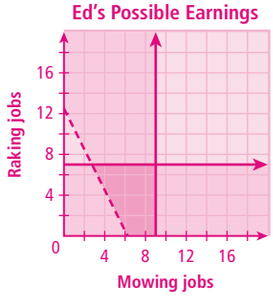
Additional Examples

Example 4

In one week, Ed can mow at most 9 times and rake at most 7 times. He charges $20 for mowing and $10 for raking. He needs to earn more than $125 in one week. Show and describe all the possible combinations of mowing and raking that Ed can do to meet his goal. List two possible combinations.



Reasonable answers must be whole numbers. Possible answer: (7 mowing, 4 raking), (8 mowing, 1 raking)

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- When do you need to check only one inequality? Why?

EXAMPLES 2–3

- How can you check that you shaded the correct areas?

EXAMPLE 4

- How do you choose which region to select the combinations from?

3 Close

Summarize

Emphasize that an ordered pair is a solution of a system of inequalities only if it makes every inequality in that system true. Graphically, the solutions will be in the overlapping shaded regions. Remind students that points on a boundary line are solutions if the line is solid, and are not solutions if the line is dashed. The intersection of boundary lines is a solution only if both lines are solid.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

Possible answers:

1. To write a system of linear inequalities from a graph, write the linear inequality for each of the graphs that make up the system.
2. See Additional Answers.

5-6 Exercises

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Homework Help Online
Parent Resources Online

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
- **Basic** 16–22, 34, 35
- **Average** 16–22, 34, 35
- **Advanced** 16–22, 34, 35

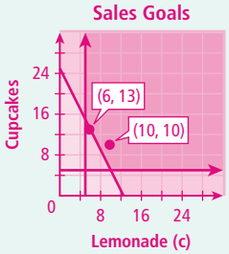
If you finished Examples **1–4**
- **Basic** 16–29, 30–38 even, 39, 42, 44–46
- **Average** 16–29, 30–42 even, 44–47
- **Advanced** 16–29, 30, 41–50

Homework Quick Check
Quickly check key concepts.
Exercises: 16, 22, 26, 29, 42

Answers

5–14. See Additional Answers.

15.



Possible answer:
(6 c lemonade, 13 cupcakes),
(10 c lemonade, 10 cupcakes)

19–28. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 15, 29–30, 38, 40, 42

Construct viable arguments and critique the reasoning of others. Exercises 39, 41, 43

Attend to precision. Exercise 47

Look for and make use of structure. Exercises 48, 50

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GUIDED PRACTICE

1. **Vocabulary** A solution of a system of inequalities is a solution of _____?_____ of the inequalities in the system. (*at least one* or *all*) **all**

SEE EXAMPLE 1

Tell whether the ordered pair is a solution of the given system.

2. $(0, 0)$; $\begin{cases} y < -x + 3 \\ y < x + 2 \end{cases}$ **yes**
3. $(0, 0)$; $\begin{cases} y < 3 \\ y > x - 2 \end{cases}$ **yes**
4. $(1, 0)$; $\begin{cases} y > 3x \\ y \leq x + 1 \end{cases}$ **no**

SEE EXAMPLE 2

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

5. $\begin{cases} y < 2x - 1 \\ y > 2 \end{cases}$
6. $\begin{cases} x < 3 \\ y > x - 2 \end{cases}$
7. $\begin{cases} y \geq 3x \\ 3x + y \geq 3 \end{cases}$
8. $\begin{cases} 2x - 4y \leq 8 \\ y > x - 2 \end{cases}$

SEE EXAMPLE 3

Graph each system of linear inequalities. Describe the solutions.

9. $\begin{cases} y > 2x + 3 \\ y < 2x \end{cases}$
10. $\begin{cases} y \leq -3x - 1 \\ y \geq -3x + 1 \end{cases}$
11. $\begin{cases} y > 4x - 1 \\ y \leq 4x + 1 \end{cases}$
12. $\begin{cases} y < -x + 3 \\ y > -x + 2 \end{cases}$
13. $\begin{cases} y > 2x - 1 \\ y > 2x - 4 \end{cases}$
14. $\begin{cases} y \leq -3x + 4 \\ y \leq -3x - 3 \end{cases}$

SEE EXAMPLE 4

15. **Business** Sandy makes \$2 profit on every cup of lemonade that she sells and \$1 on every cupcake that she sells. Sandy wants to sell at least 5 cups of lemonade and at least 5 cupcakes per day. She wants to earn at least \$25 per day. Show and describe all the possible combinations of lemonade and cupcakes that Sandy needs to sell to meet her goals. List two possible combinations.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–18	1
19–22	2
23–28	3
29	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Tell whether the ordered pair is a solution of the given system.

16. $(0, 0)$; $\begin{cases} y > -x - 1 \\ y < 2x + 4 \end{cases}$ **yes**
17. $(0, 0)$; $\begin{cases} x + y < 3 \\ y > 3x - 4 \end{cases}$ **yes**
18. $(1, 0)$; $\begin{cases} y > 3x \\ y > 3x + 1 \end{cases}$ **no**

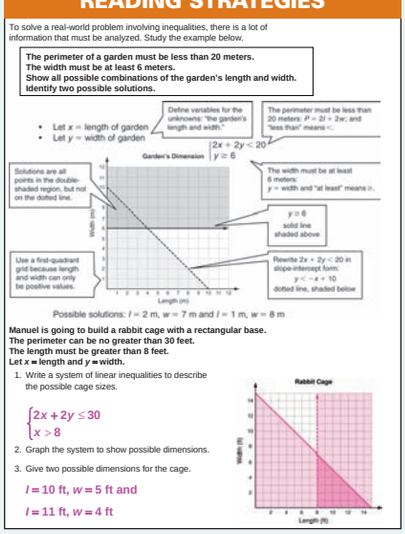
Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

19. $\begin{cases} y < -3x - 3 \\ y \geq 0 \end{cases}$
20. $\begin{cases} y < -1 \\ y > 2x - 1 \end{cases}$
21. $\begin{cases} y > 2x + 4 \\ 6x + 2y \geq -2 \end{cases}$
22. $\begin{cases} 9x + 3y \leq 6 \\ y > x \end{cases}$

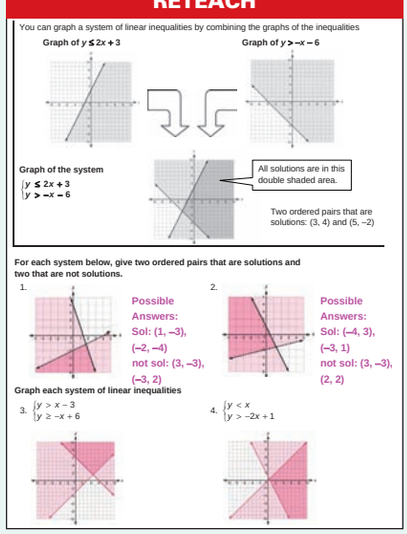
Graph each system of linear inequalities. Describe the solutions.

23. $\begin{cases} y < 3 \\ y > 5 \end{cases}$
24. $\begin{cases} y < x - 1 \\ y > x - 2 \end{cases}$
25. $\begin{cases} x \geq 2 \\ x \leq 2 \end{cases}$
26. $\begin{cases} y > -4x - 3 \\ y < -4x + 2 \end{cases}$
27. $\begin{cases} y > -1 \\ y > 2 \end{cases}$
28. $\begin{cases} y \leq 2x + 1 \\ y \leq 2x - 4 \end{cases}$

READING STRATEGIES



RETEACH



29. **Multi-Step** Linda works at a pharmacy for $15 an hour. She also baby-sits for $10 an hour. Linda needs to earn at least $90 per week, but she does not want to work more than 20 hours per week. Show and describe the number of hours Linda could work at each job to meet her goals. List two possible solutions.

30. **Farming** Tony wants to plant at least 40 acres of corn and at least 50 acres of soybeans. He wants no more than 200 acres of corn and soybeans. Show and describe all the possible combinations of the number of acres of corn and of soybeans Tony could plant. List two possible combinations.

Graph each system of linear inequalities.

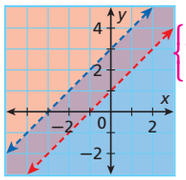
31. $\begin{cases} y \geq -3 \\ y \geq 2 \end{cases}$

32. $\begin{cases} y > -2x - 1 \\ y > -2x - 3 \end{cases}$

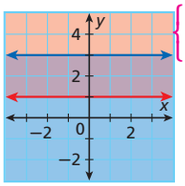
33. $\begin{cases} x \leq -3 \\ x \geq 1 \end{cases}$

34. $\begin{cases} y < 4 \\ y > 0 \end{cases}$

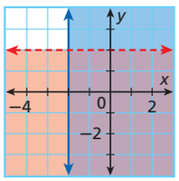
Write a system of linear inequalities to represent each graph.

35. 

$\begin{cases} y > x + 1 \\ y < x + 3 \end{cases}$

36. 

$\begin{cases} y \geq 1 \\ y \leq 3 \end{cases}$

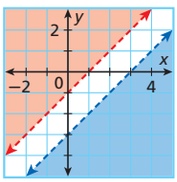
37. 

$\begin{cases} y < 2 \\ x \geq -2 \end{cases}$

In 1959, the first class graduated from the Air Force Academy. The first class that included women graduated in 1980.

38. **Military** For males to enter the United States Air Force Academy, located in Colorado Springs, CO, they must be at least 17 but less than 23 years of age. Their standing height must be not less than 60 inches and not greater than 80 inches. Graph all possible heights and ages for eligible male candidates. Give three possible combinations.

39. ///ERROR ANALYSIS/// Two students wrote a system of linear inequalities to describe the graph. Which student is incorrect? Explain the error. **Student B; the inequality symbols are incorrect.**



A $\begin{cases} y < x - 3 \\ y > x - 1 \end{cases}$

B $\begin{cases} y > x - 3 \\ y < x - 1 \end{cases}$

40. **Recreation** Vance wants to fence in a rectangular area for his dog. He wants the length of the rectangle to be at least 30 feet and the perimeter to be no more than 150 feet. Graph all possible dimensions of the rectangle.

41. **Critical Thinking** Can the solutions of a system of linear inequalities be the points on a line? Explain.

42. Gloria is starting her own company making teddy bears. She has enough bear bodies to create 40 bears. She will make girl bears and boy bears.
   a. Write an inequality to show this situation.
   b. Gloria will charge $15 for girl bears and $12 for boy bears. She wants to earn at least $540 a week. Write an inequality to describe this situation.
   c. Graph this situation and locate the solution region.

(cl), © Reuters/CORBIS; (bl), Nathan Kaiy/HMH

COMMON ERROR ALERT

Students may have trouble graphing systems containing horizontal or vertical boundary lines, such as in **Exercises 5** and **6.** Remind them that the line intersects the *x*-axis when *x* is the only variable in the inequality and that the line intersects the *y*-axis when *y* is the only variable.

MULTI-STEP TEST PREP **Exercise 42** involves graphing solutions of linear inequalities to determine the number of items that need to be sold to meet earning goals. This exercise prepares students for the Multi-Step Test Prep.

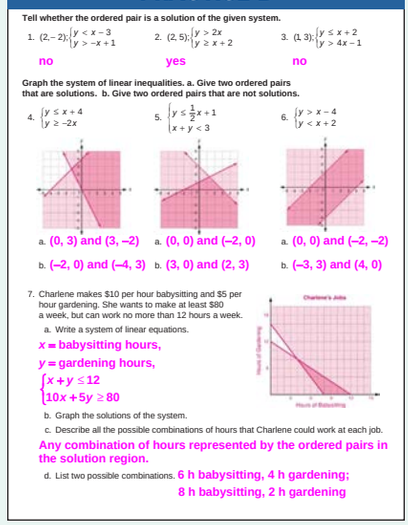
Answers

29.

Possible answer: (0 h at pharmacy, 9 h babysitting), (8.5 h at pharmacy, 10 h babysitting)

30–34, 38, 40–42. See Additional Answers.

PRACTICE A
PRACTICE C
PRACTICE B


Tell whether the ordered pair is a solution of the given system.

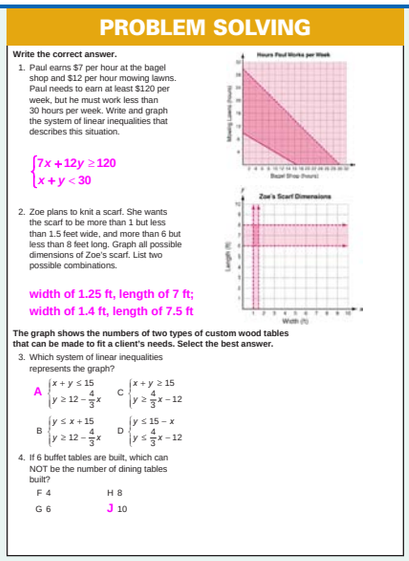
1. (2, −2); $\begin{cases} y < x - 3 \\ y > -x + 1 \end{cases}$ no
2. (2, 5); $\begin{cases} y > 2x \\ y \geq x + 2 \end{cases}$ yes
3. (1, 3); $\begin{cases} y \leq x + 2 \\ y > 4x - 1 \end{cases}$ no

Graph the system of linear inequalities. a. Give two ordered pairs that are solutions. b. Give two ordered pairs that are not solutions.

4. $\begin{cases} y \leq x + 4 \\ y \geq -2x \end{cases}$ a. (0, 3) and (3, −2) b. (−2, 0) and (−4, 3)
5. $\begin{cases} y \leq \frac{1}{2}x + 1 \\ x + y < 3 \end{cases}$ a. (0, 0) and (−2, 0) b. (3, 0) and (2, 3)
6. $\begin{cases} y > x - 4 \\ y < x + 2 \end{cases}$ a. (0, 0) and (−2, −2) b. (−3, 3) and (4, 0)

7. Charlene makes $10 per hour babysitting and $5 per hour gardening. She wants to make at least $80 a week, but can work no more than 12 hours a week.
   a. Write a system of linear equations.
   x = babysitting hours,
   y = gardening hours,
   $\begin{cases} x + y \leq 12 \\ 10x + 5y \geq 80 \end{cases}$
   b. Graph the solutions of the system.
   c. Describe all the possible combinations of hours that Charlene could work at each job.
   Any combination of hours represented by the ordered pairs in the solution region.
   d. List two possible combinations. 6 h babysitting, 4 h gardening; 8 h babysitting, 2 h gardening

PROBLEM SOLVING

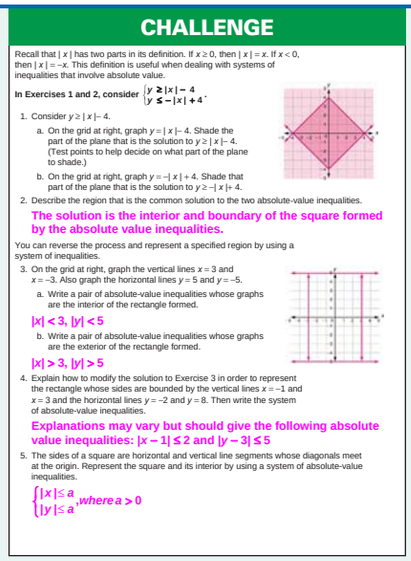

Write the correct answer.

1. Paul earns $7 per hour at the bagel shop and $12 per hour mowing lawns. Paul needs to earn at least $120 per week, but he must work less than 30 hours per week. Write and graph the system of linear inequalities that describes this situation.
   $\begin{cases} 7x + 12y \geq 120 \\ x + y < 30 \end{cases}$
2. Zoe plans to knit a scarf. She wants the scarf to be more than 1 but less than 1.5 feet wide, and more than 6 but less than 8 feet long. Graph all possible dimensions of Zoe's scarf. List two possible combinations.
   width of 1.25 ft, length of 7 ft; width of 1.4 ft, length of 7.5 ft

The graph shows the numbers of two types of custom wood tables that can be made to fit a client's needs. Select the best answer.

3. Which system of linear inequalities represents the graph?
   A $\begin{cases} x + y \leq 15 \\ y \geq 12 - \frac{4}{3}x \end{cases}$ C $\begin{cases} x + y \geq 15 \\ y \geq \frac{4}{3}x - 12 \end{cases}$
   B $\begin{cases} y \leq x + 15 \\ y \geq 12 - \frac{4}{3}x \end{cases}$ D $\begin{cases} y \leq 15 - x \\ y \leq \frac{4}{3}x - 12 \end{cases}$
4. If 6 buffet tables are built, which can NOT be the number of dining tables built?
   F 4 H 8
   G 6 J 10

CHALLENGE


Recall that $|x|$ has two parts in its definition. If $x \geq 0$, then $|x| = x$. If $x < 0$, then $|x| = -x$. This definition is useful when dealing with systems of inequalities that involve absolute value.

In Exercises 1 and 2, consider $\begin{cases} y \geq |x| - 4 \\ y \leq -|x| + 4 \end{cases}$.

1. Consider $y \geq |x| - 4$.
   a. On the grid at right, graph $y = |x| - 4$. Shade the part of the plane that is the solution to $y \geq |x| - 4$. (Test points to help decide on what part of the plane to shade.)
   b. On the grid at right, graph $y = -|x| + 4$. Shade that part of the plane that is the solution to $y \geq -|x| + 4$.
2. Describe the region that is the common solution to the two absolute-value inequalities.
   The solution is the interior and boundary of the square formed by the absolute value inequalities.

You can reverse the process and represent a specified region by using a system of inequalities.

3. On the grid at right, graph the vertical lines $x = 3$ and $x = -3$. Also graph the horizontal lines $y = 5$ and $y = -5$.
   a. Write a pair of absolute-value inequalities whose graphs are the interior of the rectangle formed.
   $|x| < 3, |y| < 5$
   b. Write a pair of absolute-value inequalities whose graphs are the exterior of the rectangle formed.
   $|x| > 3, |y| > 5$
4. Explain how to modify the solution to Exercise 3 in order to represent the rectangle whose sides are bounded by the vertical lines $x = -1$ and $x = 3$ and the horizontal lines $y = -2$ and $y = 8$. Then write the system of absolute-value inequalities.
   Explanations may vary but should give the following absolute value inequalities: $|x - 1| \leq 2$ and $|y - 3| \leq 5$
5. The sides of a square are horizontal and vertical line segments whose diagonals meet at the origin. Represent the square and its interior by using a system of absolute-value inequalities.
   $\begin{cases} |x| \leq a \\ |y| \leq a \end{cases}$, where $a > 0$

TEST PREP DOCTOR The graph in **Exercise 45** shows solutions between parallel lines. This is not possible when the inequality symbols are both $<$ or both $>$. Choices **H** and **J** can be eliminated.

Journal

Have students describe a real-world situation that can be modeled by a system of linear inequalities in two variables and in which reasonable solutions must be whole numbers.

ALTERNATIVE ASSESSMENT

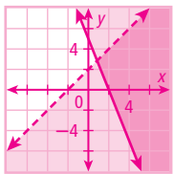
Tell students to write and graph a system of inequalities that has solutions, and identify two solutions. Then have students write and graph a system that has no solutions.

Power Presentations with PowerPoint®

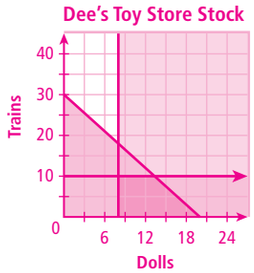
5-6 Lesson Quiz

1. Graph $\begin{cases} y < x + 2 \\ 5x + 2y \geq 10 \end{cases}$. Give two ordered pairs that are solutions and two that are not solutions.

Possible answer: solutions: (4, 4), (8, 6); not solutions: (0, 0), (−2, 3)



2. Dee has at most $150 to spend on restocking dolls and trains at her toy store. Dolls cost $7.50, and trains cost $5.00. Dee needs no more than 10 trains, and at least 8 dolls. Show and describe all possible combinations of dolls and trains that Dee can buy. List two possible combinations.



Reasonable answers must be whole numbers. Possible answer: (12 dolls, 6 trains), (16 dolls, 4 trains)

Also available online

43. Write About It What must be true of the boundary lines in a system of two linear inequalities if there is no solution of the system? Explain.

The boundary lines must be parallel. If the boundary lines are not parallel, there will be overlap.

TEST PREP

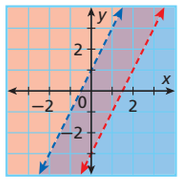
44. Which point is a solution of $\begin{cases} 2x + y \geq 3 \\ y \geq -2x + 1 \end{cases}$?

Ⓐ (0, 0) Ⓑ (0, 1) Ⓒ (1, 0) Ⓓ (1, 1)

45. Which system of inequalities best describes the graph?

Ⓕ $\begin{cases} y < 2x - 3 \\ y > 2x + 1 \end{cases}$ Ⓗ $\begin{cases} y < 2x - 3 \\ y < 2x + 1 \end{cases}$

Ⓖ $\begin{cases} y > 2x - 3 \\ y < 2x + 1 \end{cases}$ Ⓙ $\begin{cases} y > 2x - 3 \\ y > 2x + 1 \end{cases}$

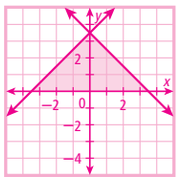


46. Short Response Graph and describe $\begin{cases} y + x > 2 \\ y \leq -3x + 4 \end{cases}$. Give two possible solutions of the system.

CHALLENGE AND EXTEND

47. Estimation Graph the given system of inequalities. Estimate the area of the overlapping solution regions. about 12 sq. units

$\begin{cases} y \geq 0 \\ y \leq x + 3.5 \\ y \leq -x + 3.5 \end{cases}$



48. Write a system of linear inequalities for which $(-1, 1)$ and $(1, 4)$ are solutions and $(0, 0)$ and $(2, -1)$ are not solutions.

49. Graph $|y| < 1$.

50. Write a system of linear inequalities for which the solutions are all the points in the third quadrant.

$\begin{cases} x < 0 \\ y < 0 \end{cases}$

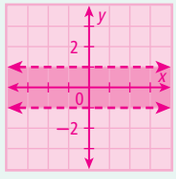
Answers

46.

Possible answer: (0, 3), (−2, 6)

48. $\begin{cases} y \geq \frac{3}{2}x + \frac{5}{2} \\ y > -\frac{1}{2}x \end{cases}$

49.



5-6 Technology LAB

Solve Systems of Linear Inequalities

A graphing calculator gives a visual solution to a system of linear inequalities.

Use with Solving Systems of Linear Inequalities

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.REI.12 Graph the … solution set to a system of linear inequalities in two variables as the intersection of the corresponding half- planes.

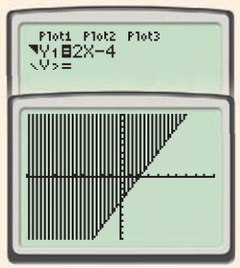
Activity

Graph the system $\begin{cases} y > 2x - 4 \\ 2.75y - x < 6 \end{cases}$. Give two ordered pairs that are solutions.

1. The first inequality is solved for y.

2. Graph the first inequality. First graph the boundary line $y = 2x - 4$. Press **Y=** and enter $2x - 4$ for **Y1.**

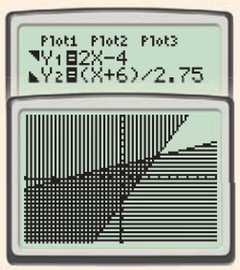
   The inequality contains the symbol $>$. The solution region is above the boundary line. Press ◀ to move the cursor to the left of **Y1.** Press **ENTER** until the icon that looks like a region above a line appears. Press **GRAPH**.



3. Solve the second inequality for y.

$$2.75y - x < 6$$
$$2.75y < x + 6$$
$$y < \frac{x + 6}{2.75}$$

4. Graph the second inequality. First graph the boundary line $y = \frac{x + 6}{2.75}$. Press **Y=** and enter $(x + 6)/2.75$ for **Y2.** The inequality contains the symbol $<$. The solution region is below the boundary line. Press ◀ to move the cursor to the left of **Y2.** Press **ENTER** until the icon that looks like a region below a line appears. Press **GRAPH**.



5. The solutions of the system are represented by the overlapping shaded regions. The points $(0, 0)$ and $(-1, 0)$ are in the shaded region.

Check Test $(0, 0)$ in both inequalities.

$y > 2x - 4$		
0	$>$	$2(0) - 4$
0	$>$	-4 ✓

$2.75y - x < 6$		
$2.75(0) - 0$	$<$	6
0	$<$	6 ✓

Test $(-1, 0)$ in both inequalities.

$y > 2x - 4$		
0	$>$	$2(-1) - 4$
0	$>$	-6 ✓

$2.75y - x < 6$		
$2.75(0) - (-1)$	$<$	6
1	$<$	6 ✓

Try This

Graph each system. Give two ordered pairs that are solutions.

1. $\begin{cases} x + 5y > -10 \\ x - y < 4 \end{cases}$
2. $\begin{cases} y > x - 2 \\ y \le x + 2 \end{cases}$
3. $\begin{cases} y > x - 2 \\ y \le 3 \end{cases}$
4. $\begin{cases} y < x - 3 \\ y - 3 > x \end{cases}$

Teacher to Teacher

It is important to point out to students that when you graph a linear inequality using a graphing calculator, the boundary line will always appear solid.

Remind students that they will need to consider the points on the boundary line and decide whether or not they are included in the solution set based on the inequality symbol rather than the appearance of the graph.

Larry Ward
Corinth, TX

Technology LAB Organizer

Use with the lesson *Solving Systems of Linear Inequalities*

Pacing:
Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Use a graphing calculator to view solutions of a system of linear inequalities.

Materials: graphing calculator

INTERACTIVE **Online Edition**
Graphing Calculator, TechKeys

Countdown

Lab Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

After the equation for the boundary line is entered, discuss how to determine whether to shade above or below the line.

Close

Key Concept

You can graph a system of inequalities on a graphing calculator by entering equations for each boundary line and graphing the solutions of each inequality.

Assessment

Journal Have students explain how to use a graphing calculator to solve a system of linear inequalities.

Answers

1–4. See Additional Answers.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half- planes.

Learn It Online
State Resources Online

SECTION 5B MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

Online Edition

Answers

2. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Learn It Online
State Resources Online

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Equations and Formulas

Bearable Sales Gloria makes teddy bears. She dresses some as girl bears with dresses and bows and some as boy bears with bow ties. She is running low on supplies. She has only 100 eyes, 30 dresses, and 60 ties that can be used as bows on the girls and bow ties on the boys.

1. Write the inequalities that describe this situation. Let x represent the number of boy bears and y represent the number of girl bears.

1. $2x + 2y \leq 100$; $y \leq 30$; $x + y \leq 60$

2. Graph the inequalities and locate the region showing the number of boy and girl bears Gloria can make.

3. List at least three combinations of girl and boy bears that Gloria can make.

3. Possible answer: (5 boys, 20 girls), (10 boys, 15 girls), (30 boys, 5 girls)

For 4 and 5, use the table.

4. Using the boundary line in your graph from Problem 2, copy and complete the table with the corresponding number of girl bears.

Bear Combinations	
Boy	Girl
0	30
10	30
20	30
30	20
40	10
50	0

5. Gloria sells the bears for profit. She makes a profit of \$8 for the girl bears and \$5 for the boy bears. Use the table from Problem 4 to find the profit she makes for each given combination.

5. \$240; \$290; \$340; \$310; \$280; \$250

6. Which combination is the most profitable? Explain. Where does it lie on the graph?

6. 20 boy bears and 30 girl bears makes a profit of \$340, which is the most; it lies at the intersection of the boundary lines $y \leq 30$ and $2x + 2y \leq 100$.

(all), Nathan Kaey/HMH

INTERVENTION

Scaffolding Questions

1. How many eyes are there per bear? 2 Which type of bear will use the dresses? girl bears Which type of bear will use the bow? both girl and boy bears
2. Can reasonable solutions be negative? no Explain. Gloria cannot sell a negative number of bears.
3. Can reasonable solutions include fractions or decimals? no Explain. Gloria cannot sell a fraction of a bear.
4. Which lines are included in the solution boundary? $y = 30$, $2x + 2y = 100$
5. What is the formula for Gloria's profit? $P = 5x + 8y$
6. How can you tell which combination is more profitable? the combination that results in the greatest value of P

Extension

What is the most profitable combination if Gloria makes a profit of \$5 for girl bears and \$8 for boy bears? 30 boy bears and 20 girl bears.

READY TO GO ON?

Quiz for Lessons 5-5 Through 5-6

5-5 Solving Linear Inequalities

Tell whether the ordered pair is a solution of the inequality.

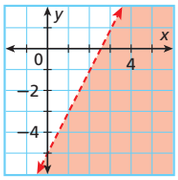
1. $(3, -2)$; $y < -2x + 1$ — no
2. $(2, 1)$; $y \geq 3x - 5$ — yes
3. $(1, -6)$; $y \leq 4x - 10$ — yes

Graph the solutions of each linear inequality.

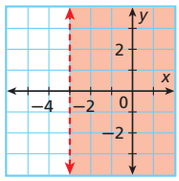
4. $y \geq 4x - 3$
5. $3x - y < 5$
6. $2x + 3y < 9$
7. $y \leq -\frac{1}{2}x$

8. Theo's mother has given him at most \$150 to buy clothes for school. The pants cost \$30 each and the shirts cost \$15 each. Write a linear inequality to describe the situation. Graph the solutions and give three combinations of pants and shirts that Theo could buy.

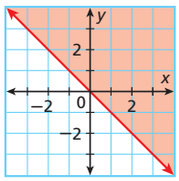
Write an inequality to represent each graph.

9. 

$y < 2x - 5$

10. 

$x > -3$

11. 

$y \geq -x$

5-6 Solving Systems of Linear Inequalities

Tell whether the ordered pair is a solution of the given system.

12. $(-3, -1)$; $\begin{cases} y > -2 \\ y < x + 4 \end{cases}$ — yes
13. $(-3, 0)$; $\begin{cases} y \leq x + 4 \\ y \geq -2x - 6 \end{cases}$ — yes
14. $(0, 0)$; $\begin{cases} y \geq 3x \\ 2x + y < -1 \end{cases}$ — no

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

15. $\begin{cases} y > -2 \\ y < x + 3 \end{cases}$
16. $\begin{cases} x + y \leq 2 \\ 2x + y \geq -1 \end{cases}$
17. $\begin{cases} 2x - 5y \leq -5 \\ 3x + 2y < 10 \end{cases}$

Graph each system of linear inequalities. Describe the solutions.

18. $\begin{cases} y \geq x + 1 \\ y \geq x - 4 \end{cases}$
19. $\begin{cases} y \geq 2x - 1 \\ y < 2x - 3 \end{cases}$
20. $\begin{cases} y < -3x + 5 \\ y > -3x - 2 \end{cases}$

21. A grocer sells mangos for \$4/lb and apples for \$3/lb. The grocer starts with 45 lb of mangos and 50 lb of apples each day. The grocer's goal is to make at least \$300 by selling mangos and apples each day. Show and describe all possible combinations of mangos and apples that could be sold to meet the goal. List two possible combinations.

18. The solutions of the system are the solutions of $y \geq x + 1$.
19. The system has no solutions.
20. The solutions are between the graphs of $y = -3x + 5$ and $y = -3x - 2$.

READY TO GO ON? SECTION 5B

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

Answers

4–8, 15–17, 21. See Additional Answers.

READY TO GO ON?
Diagnose and Prescribe

NO INTERVENE

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 5B		
	Worksheets	Online	
Lesson 5-5	5-5 Intervention	Activity 5-5	Diagnose and Prescribe Online
Lesson 5-6	5-6 Intervention	Activity 5-6	

YES ENRICH

***Ready to Go On? Enrichment*, Section 5B**
Worksheets
Online

Organizer

Objective: Help students organize and review key concepts and skills presented in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Lesson Tutorial Videos

Answers

1. independent system
2. system of linear equations
3. solution of a system of linear inequalities
4. inconsistent system
5. independent system
6. no
7. yes
8. yes
9. no
10. $(-1, -1)$
11. $(3, 4)$
12. 8 h; \$10

CHAPTER 5 Study Guide: Review

Vocabulary

consistent system
dependent system
inconsistent system
independent system
linear inequality
solution of a linear inequality
solution of a system of linear equations
solutions of a system of linear inequalities
system of linear equations
system of linear inequalities

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is a system that has exactly one solution.
2. A set of two or more linear equations that contain the same variable(s) is a(n) ___?___.
3. The ___?___ consists of all the ordered pairs that satisfy all the inequalities in the system.
4. A system consisting of equations of parallel lines with different y-intercepts is a(n) ___?___.
5. A(n) ___?___ consists of two intersecting lines.

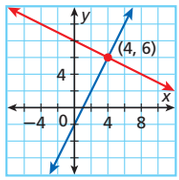
5-1 Solving Systems by Graphing

EXAMPLE

■ **Solve** $\begin{cases} y = 2x - 2 \\ x + 2y = 16 \end{cases}$ **by graphing.**

Check your answer.

$\begin{cases} y = 2x - 2 \\ y = -\frac{1}{2}x + 8 \end{cases}$ *Write the second equation in slope-intercept form.*



The solution appears to be at $(4, 6)$.

$y = 2x - 2$	
6	$2(4) - 2$
6	6 ✓

$x + 2y = 16$	
$4 + 2(6)$	16
16	16 ✓

The ordered pair $(4, 6)$ makes both equations true, so it is a solution of the system.

EXERCISES

Tell whether the ordered pair is a solution of the given system.

6. $(0, -5)$; $\begin{cases} y = -6x + 5 \\ x - y = 5 \end{cases}$
7. $(4, 3)$; $\begin{cases} x - 2y = -2 \\ y = \frac{1}{2}x + 1 \end{cases}$
8. $\left(1\frac{3}{4}, 7\frac{1}{4}\right)$; $\begin{cases} x + y = 9 \\ 2y = 6x + 4 \end{cases}$
9. $(-1, -1)$; $\begin{cases} y = -2x + 5 \\ 3y = 6x + 3 \end{cases}$

Solve each system by graphing. Check your answer.

10. $\begin{cases} y = 3x + 2 \\ y = -2x - 3 \end{cases}$
11. $\begin{cases} y = -\frac{1}{3}x + 5 \\ 2x - 2y = -2 \end{cases}$
12. Raheel is comparing the cost of two parking garages. Garage A charges a flat fee of \$6 per car plus \$0.50 per hour. Garage B charges a flat fee of \$2 per car plus \$1 per hour. After how many hours will the cost at garage A be the same as the cost at garage B? What will that cost be?

5-2 Solving Systems by Substitution

EXAMPLE

■ Solve $\begin{cases} 2x - 3y = -2 \\ y - 3x = 10 \end{cases}$ by substitution.

Step 1 $y - 3x = 10$ — *Solve the second equation for y.*

$y = 3x + 10$

Step 2 $2x - 3y = -2$ — *Substitute $3x + 10$ for y in the first equation.*

$2x - 3(3x + 10) = -2$

Step 3 $2x - 9x - 30 = -2$ — *Solve for x.*

$-7x - 30 = -2$

$-7x = 28$

$x = -4$

Step 4 $y - 3x = 10$ — *Substitute -4 for x.*

$y - 3(-4) = 10$

$y + 12 = 10$ — *Find the value of y.*

$y = -2$

Step 5 $(-4, -2)$ — *Write the solution as an ordered pair.*

To check the solution, substitute $(-4, -2)$ into both equations in the system.

EXERCISES

Solve each system by substitution.

13. $\begin{cases} y = x + 3 \\ y = 2x + 12 \end{cases}$

14. $\begin{cases} y = -4x \\ y = 2x - 3 \end{cases}$

15. $\begin{cases} 2x + y = 4 \\ 3x + y = 3 \end{cases}$

16. $\begin{cases} x + y = -1 \\ y = -2x + 3 \end{cases}$

17. $\begin{cases} x = y - 7 \\ -y - 2x = 8 \end{cases}$

18. $\begin{cases} \frac{1}{2}x + y = 9 \\ 3x - 4y = -6 \end{cases}$

19. The Nash family's car needs repairs. Estimates for parts and labor from two garages are shown below.

Garage	Parts ($)	Labor ($ per hour)
Motor Works	650	70
Jim's Car Care	800	55

For how many hours of labor will the total cost of fixing the car be the same at both garages? What will that cost be? Which garage will be cheaper if the repairs require 8 hours of labor? Explain.

5-3 Solving Systems by Elimination

EXAMPLE

■ Solve $\begin{cases} 2x - 3y = -8 \\ x + 4y = 7 \end{cases}$ by elimination.

Step 1 *Multiply the second equation by -2.*

$2x - 3y = -8$

$+(-2)(x + 4y = 7)$

Eliminate x.

$2x - 3y = -8$

$+(-2x - 8y = -14)$

Step 2 $-11y = -22$ — *Solve for y.*

$y = 2$

Step 3 $2x - 3y = -8$

$2x - 3(2) = -8$ — *Substitute 2 for y.*

$2x - 6 = -8$ — *Simplify and solve for x.*

$2x = -2$

$x = -1$

Step 4 $(-1, 2)$ — *Write the solution as an ordered pair.*

To check the solution, substitute $(-1, 2)$ into both equations in the system.

EXERCISES

Solve each system by elimination.

20. $\begin{cases} 4x + y = -1 \\ 2x - y = -5 \end{cases}$

21. $\begin{cases} x + 2y = -1 \\ x + y = 2 \end{cases}$

22. $\begin{cases} x + y = 12 \\ 2x + 5y = 27 \end{cases}$

23. $\begin{cases} 3x - 2y = -6 \\ \frac{1}{3}x + 3y = 9 \end{cases}$

Solve each system by any method. Explain why you chose each method. Check your answer.

24. $\begin{cases} 3x + y = 2 \\ y = -4x \end{cases}$

25. $\begin{cases} y = \frac{1}{3}x - 6 \\ y = -2x + 1 \end{cases}$

26. $\begin{cases} 2y = -3x \\ y = -2x + 2 \end{cases}$

27. $\begin{cases} x - y = 0 \\ 3x + y = 8 \end{cases}$

Answers

13. $(-9, -6)$

14. $\left(\frac{1}{2}, -2\right)$

15. $(-1, 6)$

16. $(4, -5)$

17. $(-5, 2)$

18. $(6, 6)$

19. 10 h; \$1350; Motor Works; 8 hours will cost \$30 less at Motor Works.

20. $(-1, 3)$

21. $(5, -3)$

22. $(11, 1)$

23. $(0, 3)$

24. $(-2, 8)$; possible answer: substitution; the second equation is already solved for *y*, and *y* has a coefficient of 1 in the first equation.

25. $(3, -5)$; possible answer: graphing; both equations are already in slope-intercept form.

26. $(4, -6)$; possible answer: substitution; the second equation is already solved for *y*.

27. $(2, 2)$; possible answer: elimination; the coefficients of the *y*-terms are opposites.

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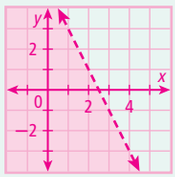
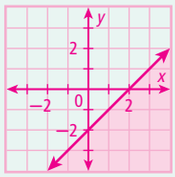
Answers

28. no solution
29. infinitely many solutions
30. $(-2, -4)$
31. infinitely many solutions
32. infinitely many solutions
33. $(-1, -3)$
34. Yes; at 7 months, Tristan and Marco will have the same number of DVDs.
35. consistent, independent; one solution
36. inconsistent; no solution
37. consistent, dependent; infinitely many solutions
38. inconsistent; no solution
39. consistent, independent; one solution
40. consistent, dependent; infinitely many solutions
41. inconsistent; no solution
42. no
43. yes
44. yes
45. no
46. 

47. 

5-4 Solving Special Systems

EXAMPLES

Classify each system. Give the number of solutions.

■ $\begin{cases} y = 3x + 4 \\ 6x - 2y = -8 \end{cases}$

Use the substitution method because the first equation is solved for y.

$6x - 2(3x + 4) = -8$ *Substitute $3x + 4$ for y in the second equation.*

$6x - 6x - 8 = -8$

$-8 = -8$ ✓ *True*

The equation is an identity. There are infinitely many solutions.

This system is **consistent** and **dependent.** When graphed, the two lines are coincident (the same line)—they have identical slopes and y-intercepts.

■ $\begin{cases} y = 2x - 1 \\ 2x - y = -2 \end{cases}$

Compare slopes and y-intercepts. Write both equations in slope-intercept form.

$\begin{cases} y = 2x - 1 & \Rightarrow y = 2x - 1 \\ 2x - y = -2 & \Rightarrow y = 2x + 2 \end{cases}$

The lines have the same slope and different y-intercepts. The lines are parallel.

The lines never intersect, so this system is **inconsistent**. It has **no solution.**

■ $\begin{cases} 2x - y = 6 \\ y = x - 1 \end{cases}$

Write both equations in slope-intercept form.

$\begin{cases} 2x - y = 6 & \Rightarrow y = 2x - 6 \\ y = x - 1 & \Rightarrow y = 1x - 1 \end{cases}$

The lines intersect because they have different slopes.

The system is **consistent** and **independent.** There is **one solution:** $(5, 4)$.

EXERCISES

Solve each system of linear equations.

28. $\begin{cases} y = \frac{1}{4}x - 3 \\ y = \frac{1}{4}x + 5 \end{cases}$

29. $\begin{cases} y = -x + 4 \\ x + y = 4 \end{cases}$

30. $\begin{cases} y = 3x + 2 \\ y = 2x \end{cases}$

31. $\begin{cases} -4x - y = 6 \\ \frac{1}{2}y = -2x - 3 \end{cases}$

32. $\begin{cases} x + 2y = 8 \\ y = -\frac{1}{2}x + 4 \end{cases}$

33. $\begin{cases} y - 2x = -1 \\ y + 2x = -5 \end{cases}$

34. Tristan and his friend Marco just started DVD collections. They continue to get DVDs at the rate shown in the table below. Will Tristan ever have the same number of DVDs as Marco? Explain.

DVD Collections			
	Month 1	Month 2	Month 3
Tristan	2	7	12
Marco	8	12	16

Classify each system. Give the number of solutions.

35. $\begin{cases} y = \frac{1}{2}x + 2 \\ y = \frac{1}{4}x - 8 \end{cases}$

36. $\begin{cases} y = 3x - 7 \\ y = 3x + 2 \end{cases}$

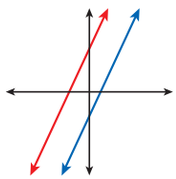
37. $\begin{cases} 2x + y = 2 \\ y - 2 = -2x \end{cases}$

38. $\begin{cases} -3x - y = -5 \\ y = -3x - 5 \end{cases}$

39. $\begin{cases} 2x + 3y = 1 \\ 3x + 2y = 1 \end{cases}$

40. $\begin{cases} x + \frac{1}{2}y = 3 \\ 2x = 6 - y \end{cases}$

41. The two parallel lines graphed below represent a system of equations. Classify the system and give the number of solutions.



5-5 Solving Linear Inequalities

EXAMPLE

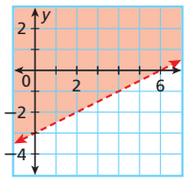
■ **Graph the solutions of $x - 2y < 6$.**

Step 1 Solve the inequality for y.

$$x - 2y < 6$$
$$-2y < -x + 6$$
$$y > \frac{1}{2}x - 3$$

Step 2 Graph $y = \frac{1}{2}x - 3$. Use a dashed line for $>$.

Step 3 The inequality is $>$, so shade above the boundary line.



Check Substitute $(0, 0)$ for (x, y).

$x - 2y$	< 6
$0 - 2(0)$	6
0	< 6 ✓

(0, 0) satisfies the inequality, so the graph is shaded correctly.

EXERCISES

Tell whether the ordered pair is a solution of the inequality.

42. $(0, -3)$; $y < 2x - 3$

43. $(2, -1)$; $y \geq x - 3$

44. $(6, 0)$; $y > -3x + 4$

45. $(10, 10)$; $y \leq x - 3$

Graph the solutions of each linear inequality.

46. $y < -2x + 5$

47. $x - y \geq 2$

48. $-x + 2y \geq 6$

49. $y > -4x$

50. $x + y + 4 > 0$

51. $5 - y \geq 2x$

52. The Mathematics Club is selling pizza and lemonade to raise money for a trip. They estimate that the trip will cost at least \$450. They earn \$2 on each slice of pizza and \$1 on each bottle of lemonade. Write an inequality to describe the situation. Graph the solutions and then give two combinations of the number of pizza slices and number of lemonade bottles they could sell to meet their goal.

5-6 Solving Systems of Linear Inequalities

EXAMPLES

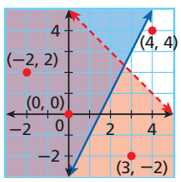
■ **Graph** $\begin{cases} y < -x + 5 \\ y \geq 2x - 3 \end{cases}$**. Give two ordered pairs that are solutions and two that are not solutions.**

Graph both inequalities.

The solutions of the system are represented by the overlapping shaded regions.

The points $(0, 0)$ and $(-2, 2)$ are solutions of the system.

The points $(3, -2)$ and $(4, 4)$ are not solutions.



EXERCISES

Tell whether the ordered pair is a solution of the given system.

53. $(3, 3)$; $\begin{cases} y > -2x + 9 \\ y \geq x \end{cases}$

54. $(-1, 0)$; $\begin{cases} 2x - y > -5 \\ y \leq -3x - 3 \end{cases}$

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

55. $\begin{cases} y \geq x + 4 \\ y > 6x - 3 \end{cases}$

56. $\begin{cases} y \leq -2x + 8 \\ y > 3x - 5 \end{cases}$

57. $\begin{cases} -x + 2y > 6 \\ x + y < 4 \end{cases}$

58. $\begin{cases} x - y > 7 \\ x + 3y \leq 15 \end{cases}$

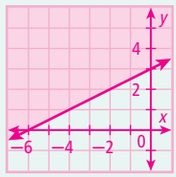
Graph each system of linear inequalities.

59. $\begin{cases} y > -x - 6 \\ y < -x + 5 \end{cases}$

60. $\begin{cases} 4x + 2y \geq 10 \\ 6x + 3y < -9 \end{cases}$

Answers

48.



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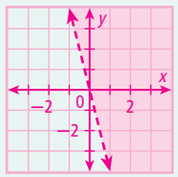

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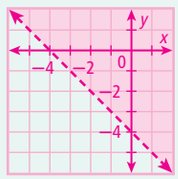

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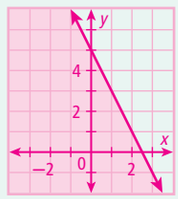
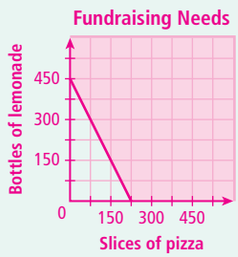


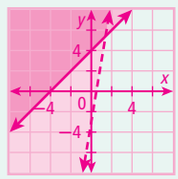
52. Let x = slices of pizza, and y = bottles of soda; $2x + y \geq 450$; possible answer: (200 slices, 50 bottles), (150 slices, 150 bottles).



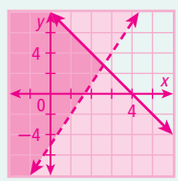
53. no

54. yes

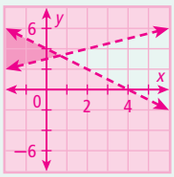
55. Possible answer: solutions: $(-6, 6)$, $(-10, 0)$; not solutions: $(0, 0)$, $(4, -4)$



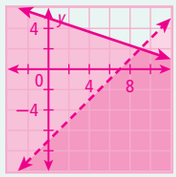
56. Possible answer: solutions: $(0, 0)$, $(-5, 0)$; not solutions: $(8, 0)$, $(3, -3)$



57. Possible answer: solutions: $(-6, 2)$, $(-8, 1)$; not solutions: $(0, 0)$, $(4, 1)$



58. Possible answer: solutions: $(8, -8)$, $(9, 0)$; not solutions: $(0, 0)$, $(0, -4)$



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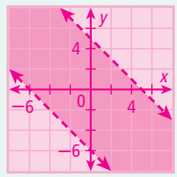

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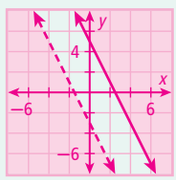


CHAPTER 5 CHAPTER TEST

Organizer

Objective: Assess students' mastery of concepts and skills in this chapter.

Online Edition

Resources

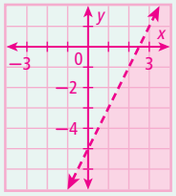
Assessment Resources

Chapter Tests

- Free Response (Levels A, B, C)
- Multiple Choice (Levels A, B, C)
- Performance Assessment

Answers

17.



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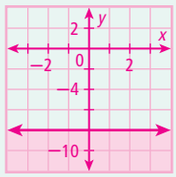

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CHAPTER 5 CHAPTER TEST

Tell whether the ordered pair is a solution of the given system.

1. $(1, -4)$; $\begin{cases} y = -4x \\ y = 2x - 2 \end{cases}$ **no**

2. $(0, -1)$; $\begin{cases} 3x - y = 1 \\ x + 5y = -5 \end{cases}$ **yes**

3. $(3, 2)$; $\begin{cases} x - 2y = -1 \\ -3x + 2y = 5 \end{cases}$ **no**

Solve each system by graphing.

4. $\begin{cases} y = x - 3 \\ y = -2x - 3 \end{cases}$ **(0, −3)**

5. $\begin{cases} 2x + y = -8 \\ y = \frac{1}{3}x - 1 \end{cases}$ **(−3, −2)**

6. $\begin{cases} y = -x + 4 \\ x = y + 2 \end{cases}$ **(3, 1)**

Solve each system by substitution.

7. $\begin{cases} y = -6 \\ y = -2x - 2 \end{cases}$ **(2, −6)**

8. $\begin{cases} -x + y = -4 \\ y = 2x - 11 \end{cases}$ **(7, 3)**

9. $\begin{cases} x - 3y = 3 \\ 2x = 3y \end{cases}$ **(−3, −2)**

10. The costs for services at two kennels are shown in the table. Joslyn plans to board her dog and have him bathed once during his stay. For what number of days will the cost for boarding and bathing her dog at each kennel be the same? What will that cost be? If Joslyn plans a week-long vacation, which is the cheaper service? Explain. **6; $195; Fido's; it will cost less per day after 6 days.**

Kennel Costs		
	Boarding ($ per day)	Bathing ($)
Pet Care	30	15
Fido's	28	27

Solve each system by elimination.

11. $\begin{cases} 3x - y = 7 \\ 2x + y = 3 \end{cases}$ **(2, −1)**

12. $\begin{cases} 4x + y = 0 \\ x + y = -3 \end{cases}$ **(1, −4)**

13. $\begin{cases} 2x + y = 3 \\ x - 2y = -1 \end{cases}$ **(1, 1)**

Classify each system. Give the number of solutions.

14. $\begin{cases} y = 6x - 1 \\ 6x - y = 1 \end{cases}$ **cons. and dep.; inf. many solutions**

15. $\begin{cases} y = -3x - 3 \\ 3x + y = 3 \end{cases}$ **incons.; no sol.**

16. $\begin{cases} 2x - y = 1 \\ -4x + y = 1 \end{cases}$ **cons. and indep.; one sol.**

Graph the solutions of each linear inequality.

17. $y < 2x - 5$

18. $-y \geq 8$

19. $y > \frac{1}{3}x$

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

20. $\begin{cases} y > \frac{1}{2}x - 5 \\ y \leq 4x - 1 \end{cases}$

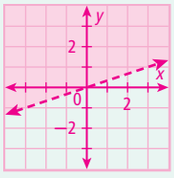
21. $\begin{cases} y > -x + 4 \\ 3x - y > 3 \end{cases}$

22. $\begin{cases} y \geq 2x \\ y - 2x < 6 \end{cases}$

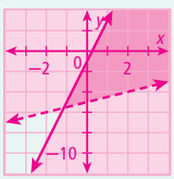
23. Ezra and Tava sold at least 150 coupon books. Ezra sold at most 30 books more than twice the number Tava sold. Show and describe all possible combinations of the numbers of coupon books Ezra and Tava sold. List two possible combinations.

Answers

19.

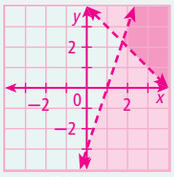


20.



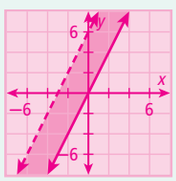
Possible answers: solutions: (2, 0), (4, 0); not solutions: (−2, 0), (−4, 0)

21.



Possible answers: solutions: (6, 4), (9, 0); not solutions: (2, −4), (0, 0)

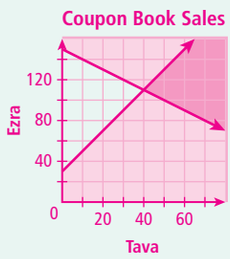
22.



Possible answers: solutions: (0, 1), (0, 2); not solutions: (−6, 0), (4, 1)

23. Ezra and Tava could have sold any combination represented by an ordered pair of whole numbers in the solutions region.

Possible answers: (Tava: 50, Ezra: 125), (Tava: 65, Ezra: 95)



College Entrance Exam Practice

CHAPTER 5

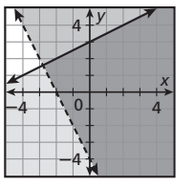
FOCUS ON ACT

Four scores are reported for the ACT Mathematics Test: one score based on all 60 problems and one for each content area. The three content areas are: Pre-Algebra/Elementary Algebra, Intermediate Algebra/Coordinate Geometry, and Plane Geometry/Trigonometry.

HOT TIP! Taking classes that cover the content areas on the ACT is a good idea. This way you will have skills from each area of the test. Preparation over a long term is better than cramming at the last minute.

You may want to time yourself as you take this practice test. It should take you about 5 minutes to complete.

1. Which system of inequalities is represented by the graph?



(A) $\begin{cases} -x + 2y < 6 \\ 2x + y > -4 \end{cases}$

(B) $\begin{cases} x - 2y \le 6 \\ 2x - y \ge 4 \end{cases}$

(C) $\begin{cases} -x + 2y \le 6 \\ 2x + y \ge 4 \end{cases}$

(D) $\begin{cases} -x + 2y \le 6 \\ 2x + y > -4 \end{cases}$

(E) $\begin{cases} x - 2y \le 6 \\ 2x - y > 4 \end{cases}$

2. What is the y-value of the solution of the given system?

$$\begin{cases} 4x + 3y = 1 \\ -4x + 3y = -7 \end{cases}$$

(F) -1

(G) 0

(H) 1

(J) 2

(K) 6

3. Wireless phone company A charges \$20 per month plus \$0.12 per minute. Wireless phone company B charges \$50 per month plus \$0.06 per minute. For how many minutes of calls will the monthly bills be the same?

(A) 80 minutes

(B) 100 minutes

(C) 160 minutes

(D) 250 minutes

(E) 500 minutes

4. Which of the following systems of equations does NOT have a solution?

(F) $\begin{cases} x + 5y = 30 \\ -4x + 5y = 10 \end{cases}$

(G) $\begin{cases} x + 5y = -30 \\ -4x + 5y = 10 \end{cases}$

(H) $\begin{cases} x + 5y = -30 \\ -4x + 5y = -10 \end{cases}$

(J) $\begin{cases} -4x + 5y = -10 \\ -8x + 10y = -20 \end{cases}$

(K) $\begin{cases} -4x + 5y = -10 \\ -4x + 5y = -30 \end{cases}$

TEST PREP DOCTOR

1. Students who chose **A** may not have assigned the appropriate inequality symbols to the dashed and solid lines. Remind students of the difference between dashed and solid lines and how they correspond to the solutions of an inequality.

2. Students who chose **H** may have found the solution for x instead of y.

3. Students who chose **A** found the value of the bill for the month, in dollars, instead of the number of minutes.

4. Students who chose **J** may not understand that coinciding lines indicate infinitely many solutions of the system. Remind students that the type of system that has no solution is represented by parallel lines.

Organizer

Objective: Provide practice for college entrance exams such as the ACT.

INTERACTIVE **Online Edition**

Resources

College Entrance Exam Practice

Questions on the ACT represent the following content areas:

Pre-Algebra, 23%
Elementary Algebra, 17%
Intermediate Algebra, 15%
Coordinate Geometry, 15%
Plane Geometry, 23%
Trigonometry, 7%

Items on this page focus on:

- Elementary Algebra
- Coordinate Geometry

CHAPTER 5 TEST TACKLER

Organizer

Objective: Provide opportunities to learn and practice common test-taking strategies.

INTERACTIVE **Online Edition**

TEST PREP DOCTOR This Test Tackler reinforces the importance of reading a test item slowly and carefully, identifying the important information, and being aware of all parts of the question. Explain to students that if they read a question too quickly, they might make assumptions that are not true, answer the wrong question, or not answer all parts of the question. Show students how to read a question for understanding by using the process shown in the example.

Any Question Type: Read the Problem for Understanding

Standardized test questions may vary in format including multiple choice, gridded response, and short or extended response. No matter what format the test uses, read each question carefully and critically. Do not rush. Be sure you completely understand what you are asked to do and what your response should include.

EXAMPLE

Extended Response

An interior decorator charges a consultation fee of $50 plus $12 per hour. Another interior decorator charges a consultation fee of $5 plus $22 per hour. Write a system of equations to find the amount of time for which the cost of both decorators will be the same. Graph the system. After how many hours will the cost be the same for both decorators? What will the cost be?

Read the problem again.

What information are you given?

the consultation fees and hourly rates of two decorators

What are you asked to do?

1. Write a system of equations.
2. Graph the system.
3. Interpret the solution to the system.

What should your response include?

1. a system of equations with variables defined
2. a graph of the system
3. the time when the cost is the same for both decorators
4. the cost at that time

Read each test item and answer the questions that follow.

Item A

Short Response Which value of b will make the lines intersect at the point $(-2, 14)$?

$$\begin{cases} y = -6x + 2 \\ y = 4x + b \end{cases}$$

1. What information are you given?
2. What are you asked to do?
3. Ming's answer to this test problem was $y = 4x + 22$. Did Ming answer correctly? Explain.

Item B

Extended Response Solve the system by using elimination. Explain how you can check your solution algebraically and graphically.

$$\begin{cases} 4x + 10y = -48 \\ 6x - 10y = 28 \end{cases}$$

4. What method does the problem ask you to use to solve the system of equations?
5. What methods does the problem ask you to use to check your solution?
6. How many parts are there to this problem? List what needs to be included in your response.

Item C

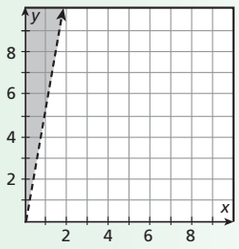
Gridded Response What is the x-coordinate of the solution to this system? $\begin{cases} y = 6x + 9 \\ y = 12x - 15 \end{cases}$

7. What question is being asked?
8. A student correctly found the solution of the system to be $(4, 33)$. What should the student mark on the grid so that the answer is correct?

HOT TIP! After you answer each item, read the item again to be sure your response includes everything that is asked for.

Item D

Short Response Write an inequality to represent the graph below. Give a real-world situation that this inequality could describe.



9. As part of his answer, a student wrote the following response:

The point (1, 5) is not a solution to the inequality because it lies on the line, but (2, 12) is a solution because it lies above the line.

Is his response appropriate? Explain.

10. What should the response include so that it answers all parts of the problem?

Item E

Multiple Choice Taylor bikes 50 miles per week and increases her distance by 2 miles each week. Josie bikes 30 miles per week and increases her distance by 10 miles each week. In how many weeks will Taylor and Josie be biking the same distance?

(A) 2.5 weeks
(B) 7.5 weeks
(C) 55 weeks
(D) 110 weeks

11. What question is being asked?
12. Carson incorrectly selected option C as his answer. What question did he most likely answer?

Answers to Test Items

A. 22
B. $(-2, -4)$
C. 4
D. $y > 5x$; Jeff opens a bank account and deposits more than \$5 a week.
E. A

TEST PREP DOCTOR In **Item A,** students are asked for the value of b, not the equation. In **Item B,** a complete answer will show three solution methods. In **Item C,** students are asked for only the x-coordinate of the solution, not the entire ordered pair. In **Item E,** students are asked for only the solution value of the independent variable, number of weeks, not the dependent variable, number of miles.

Answers

1. a system of equations and the point of intersection
2. Find the value of b that makes the system intersect at $(-2, 14)$.
3. No; Ming found the actual equation, which is correct, but the question just asked for the value of b.
4. elimination
5. algebraically and graphically
6. three parts; solving the system using elimination, checking the solution algebraically, and checking the solution graphically
7. What is the x-coordinate of the solution of the system?
8. 4
9. No; the problem did not ask anything about solutions of the inequality.
10. The response should include an inequality that represents the graph and a real-world situation that the inequality could describe.
11. In how many weeks will the girls be biking the same distance?
12. How many miles will the girls bike during the week that they are biking the same distance?

Learn It Online
State Resources Online

Organizer

Objective: Provide review and practice for this chapter and standardized tests.

Online Edition

Resources

Assessment Resources

Chapter Cumulative Test

Learn It Online
State Test Practice

Learn It Online
State Resources Online

CUMULATIVE ASSESSMENT

Multiple Choice

1. If $a > 0$ and $b < 0$, which of the following must be true?

Ⓐ $ab > 0$ Ⓒ $a + b > 0$

Ⓑ $ab < 0$ Ⓓ $a + b < 0$

2. Which of the problems below could be solved by finding the solution of this system?

$$\begin{cases} 2x + 2y = 56 \\ y = \frac{1}{3}x \end{cases}$$

Ⓕ The area of a rectangle is 56 square units. The width is one-third the length. Find the length of the rectangle.

Ⓖ The area of a rectangle is 56 square units. The length is one-third the perimeter. Find the length of the rectangle.

Ⓗ The perimeter of a rectangle is 56 units. The length is one-third more than the width. Find the length of the rectangle.

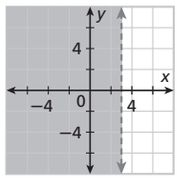
Ⓙ The perimeter of a rectangle is 56 units. The width is one-third the length. Find the length of the rectangle.

3. What is the slope of a line perpendicular to a line that passes through $(3, 8)$ and $(1, -4)$?

Ⓐ $-\frac{1}{6}$ Ⓒ 2

Ⓑ $-\frac{1}{2}$ Ⓓ 6

4. Which inequality is graphed below?



Ⓕ $-x > -3$ Ⓗ $2x < -6$

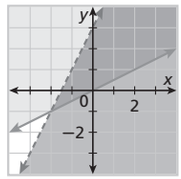
Ⓖ $-y > -3$ Ⓙ $3y < 9$

5. A chemist has a bottle of a 10% acid solution and a bottle of a 30% acid solution. He mixes the solutions together to get 500 mL of a 25% acid solution. How much of the 30% solution did he use?

Ⓐ 125 mL Ⓒ 375 mL

Ⓑ 150 mL Ⓓ 450 mL

6. Which ordered pair is NOT a solution of the system graphed below?



Ⓕ $(0, 0)$ Ⓗ $(1, 1)$

Ⓖ $(0, 3)$ Ⓙ $(2, 1)$

7. Which of the following best classifies a system of linear equations whose graph is two intersecting lines?

Ⓐ inconsistent and dependent

Ⓑ inconsistent and independent

Ⓒ consistent and dependent

Ⓓ consistent and independent

8. Which ordered pair is a solution of this system?

$$\begin{cases} 2x - y = -2 \\ \frac{1}{3}y = x \end{cases}$$

Ⓕ $(0, 2)$ Ⓗ $(2, 6)$

Ⓖ $(1, 3)$ Ⓙ $(3, 8)$

9. Where does the graph of $5x - 10y = 30$ cross the y-axis?

Ⓐ $(0, -3)$ Ⓒ $(6, 0)$

Ⓑ $\left(0, \frac{1}{2}\right)$ Ⓓ $(0, -6)$

Answers

1. B
2. J
3. A
4. F
5. C
6. G
7. D
8. H
9. A
10. F
11. C
12. 26.25
13. 2
14. 5

HOT TIP! **Most standardized tests allow you to write in your test booklet. Cross out each answer choice you eliminate. This may keep you from accidentally marking an answer other than the one you think is right. However, don't draw in your math book!**

10. Hillary needs markers and poster board for a project. The markers are \$0.79 each and the poster board is \$1.89 per sheet. She needs at least 4 sheets of poster board. Hillary has \$15 to spend on project materials. Which system models this information?

Ⓕ $\begin{cases} p \geq 4 \\ 0.79m + 1.89p \leq 15 \end{cases}$

Ⓖ $\begin{cases} 0.79m \geq 1.89p \\ 4p \leq 15 \end{cases}$

Ⓗ $\begin{cases} 4p \geq 1.89 \\ m + 4p \leq 15 \end{cases}$

Ⓙ $\begin{cases} p + m \leq 15 \\ 0.79m + 1.89p \geq 4 \end{cases}$

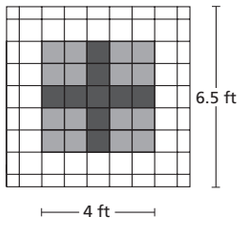
11. Find the slope of the line described by $4x - 2y = -8$.

Ⓐ -2 Ⓒ 2

Ⓑ $-\frac{1}{2}$ Ⓓ 4

Gridded Response

12. The floor in the entranceway of Kendra's house is square and measures 6.5 feet on each side. Colored tiles have been set in the center of the floor in a square measuring 4 feet on each side. The remaining floor consists of white tiles.



How many square feet of the entranceway floor consists of white tiles?

13. What value of y will make the line passing through $(4, -4)$ and $(-8, y)$ have a slope of $-\frac{1}{2}$?

14. What value of k will make the system $y - 5x = -1$ and $y = kx + 3$ inconsistent?

Short Response

15. The data in the table represents a linear relationship between x and y. Find the missing y-value. Explain your answer.

x	y
-3	-15
-1	-7
0	-3
2	
5	17

16. Graph $y > \frac{-x}{3} - 1$ on a coordinate plane. Name one point that is a solution of the inequality.

17. Marc and his brother Ty start saving money at the same time. Marc has \$145 and will add \$10 to his savings every week. Ty has \$20 and will add \$15 to his savings every week. After how many weeks will Marc and Ty have the same amount saved? What is that amount? Show your work.

18. A movie producer is looking for extras to act as office employees in his next movie. The producer needs extras that are at least 40 years old but less than 70 years old. They should be at least 60 inches tall but less than 75 inches tall. Graph all the possible combinations of ages and heights for extras that match the producer's needs. Let x represent age and y represent height. Show your work.

19. Graph the system $\begin{cases} y < -2x + 3 \\ y \geq 6x + 6. \end{cases}$

a. Is $(0, 0)$ a solution of the system you graphed? Explain why or why not.

b. Is $(-4, 5)$ a solution of the system you graphed? Explain why or why not.

Extended Response

20. Every year, Erin knits scarves and sells them at the craft fair. This year she used \$6 worth of yarn for each scarf. She also paid \$50 to rent a table at the fair. She sold every scarf for \$10.

a. Write a system of linear equations to represent the amount Erin spent and the amount she collected. Tell what your variables represent. Tell what each equation in the system represents.

b. Use any method to solve the system you wrote in part **a.** Show your work. How many scarves did Erin need to sell to make a profit? Explain.

c. Describe two ways you could check your solution to part **b.** Check your solution by using one of those ways. Show your work.

Short-Response Rubric

Items 15–19

2 Points = The student's answer is an accurate and complete execution of the task or tasks.

1 Point = The student's answer contains attributes of an appropriate response but is flawed.

0 Points = The student's answer contains no attributes of an appropriate response.

Extended Response Rubric

Item 20

4 Points = The student correctly writes a system of linear equations, identifying all variables and describes what each system represents, solves the system, and checks the answer in a method different from how it was solved.

3 Points = The student correctly writes a system of equations but does not identify any of the variables. The student solves the system and checks the answer.

2 Points = The student answers all parts correctly but does not show any work or explanation; or the student correctly answers parts **a** and **b** but does not check his or her work as stated in part **c.**

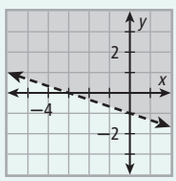
1 Point = The student writes a system of equations but does not correctly answer other parts of the problem; or the student attempts to answer all parts of the problem but does not correctly answer any part.

0 Points = The student does not answer correctly and does not attempt all parts of the problem.

Answers

15. 5; possible answer: find the slope of the line, then use a known point and the unknown point, substitute these coordinates into the slope ratio, and set it equal to the slope of the line. Then solve for y.

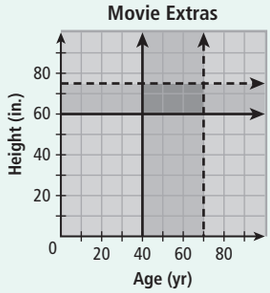
16.



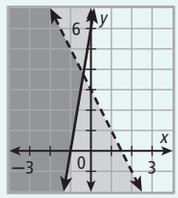
Possible answer: (3, 4)

17. 25 weeks; \$395

18.



19.



19a. No; possible answer: it is not in the overlapping region.

b. Yes; it is in the overlapping region.

20a. Let x = the number of scarves and y = the total amount of money.

$\begin{cases} y = 6x + 50 \text{ (amount Erin spent)} \\ y = 10x \text{ (amount Erin collected)} \end{cases}$

b. Possible answer: by substitution: $10x = 6x + 50$; $12.5 = x$; 13; Erin can't sell half a scarf, and 12 scarves won't make a profit.

c. Possible answer: Graph the 2 equations and find their point of intersection. Solve for y and then substitute for x and y in both equations. (Check students' work.)

Real-World CONNECTIONS

Organizer

Objective: Choose appropriate problem-solving strategies and use them to solve real-world problems.

INTERACTIVE **Online Edition**

Kayaking and Canoeing in the Pine Barrens

Reading Strategies

Have students reword **Problem 3** in their own words. Instruct them to write it so that all needed information not directly given in the problem is given and that any unnecessary information is omitted. Possible answer: A x-hour trip must be completed by 4:30 P.M. What is the latest time the trip can begin?

Using Data Using the table for **Problems 2–3**, ask students to identify where they should start if they want to end their canoe trip at Harrisville. Oswego Lake

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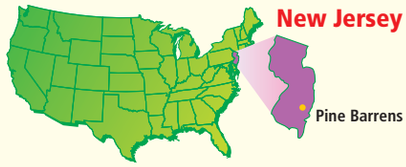


Kayaking and Canoeing in the Pine Barrens

The Pine Barrens, or Pinelands, of New Jersey cover over 1 million acres in southern and central New Jersey. Kayaking on the Wading and Oswego Rivers is a popular summertime activity. There are several campgrounds and canoe and kayak rentals located along the rivers.

Choose one or more strategies to solve each problem.

1. A group of kayakers starts at Evans Bridge and kayaks to Beaver Branch. Two other groups of kayakers start at Oswego Lake. One of these groups heads toward Harrisville, and the other heads toward Bodine Field. A fourth group starts at Speedwell and kayaks to Chips Folly. Draw a mapping diagram to represent the starting and ending points of the trips. Tell whether the relation is a function. Explain.

For 2 and 3, use the table.

2. If the average speed of canoes on the river is $1\frac{1}{2}$ miles per hour, about how far is it from Oswego Lake to Beaver Branch? **6.75 mi**

3. All river trips must be completed by 4:30 P.M. What is the latest time a canoetrip can start in order to make it from Speedwell to Evans Bridge? **10:30 A.M.**

Approximate Canoe Trip Times on the Wading and Oswego Rivers

Trip	Time (h)
Evans Bridge to Beaver Branch	2
Hawkins Bridge to Evans Bridge	3
Oswego Lake to Harrisville	4
Oswego Lake to Beaver Branch	$4\frac{1}{2}$
Hawkins Bridge to Beaver Branch	5
Speedwell to Evans Bridge	6
Speedwell to Beaver Branch	8

(bl), © Bob Krist/CORBIS; (tr), ©Bill Curtsinger/National Geographic/Getty Images

Problem-Solving Focus

Encourage students to use the four-step problem-solving process for **Problem 2.** Focus on the first and second steps: **(1) Understand the Problem:** Ask students what they are asked to find. the distance from Oswego Lake to Beaver Branch

(2) Make a Plan: Have students find two different ways to solve the problem. Use the distance formula, $d = rt$, or make a table comparing time and distance.

Answers

1.

Evans Bridge	Beaver Branch
Oswego Lake	Harrisville
Speedwell	Bodine Field
	Chips Folly

This relation is not a function because one element of the domain (Oswego Lake) is paired with more than one element of the range.

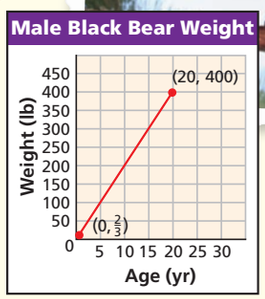
American Black Bear

American black bears are the most common bears in the United States. They can be found in 11 of the 21 counties in New Jersey. Most wild male black bears weigh between 125 and 600 pounds, while females generally weigh between 90 and 300 pounds. Their weight depends upon their age, the season of the year, and how much food is available.

Choose one or more strategies to solve each problem.

1. Black bears hibernate for about 5 months in the winter. They must store 50 to 60 pounds of fat to survive hibernation. During the month before hibernation, a black bear may consume up to 20,000 Calories per day. If a bear consumes an average of 18,500 Calories per day in the month of August, about how many total Calories will he consume that month? about 600,000 Calories

For 2, use the graph.



2. A male bear weighed $\frac{2}{3}$ pound at birth. When he died at 20 years of age, he weighed 400 pounds. Use the graph to estimate how much the bear weighed when he was 7 years old. about 150 lb
3. A white-tailed deer is running from a black bear at 20 miles per hour. It is $\frac{1}{3}$ mile in front of the bear. The bear is running at 30 miles per hour. How long will it take the bear to catch the deer? Assume both continue running at that same pace. about 2 min
4. During the hibernation months, the bear's heart rate slows to about 10 beats per minute. If the bear hibernates for 155 days, how many times will his heart beat? 2,232,000 times

(tr), Radius Images/Corbis; (br), Hope Ryden/National Geographic Image Collection

Real-World Connections

American Black Bear

Reading Strategies

ENGLISH LANGUAGE LEARNERS

In **Problem 3,** have students identify the needed information. 18,500 Calories per day for the month of August Then ask what additional information is needed to answer the question. the number of days in August

Using Data Have students explain why the graph has a segment rather than a line. Age cannot be negative, and the bear cannot live forever. Then ask why the graph does not begin at $(0, 0)$. The bear must weigh something at birth.

Problem-Solving Focus

Encourage students to use the four-step problem-solving process for **Problem 2.** Focus on the second and third steps: **(2) Make a Plan:** What equation can you write for the speed of the deer? $y = 20x + \frac{1}{3}$ for the speed of the bear? $y = 30x$ What does x represent? number of hours **(3) Solve:** Ask students how they will solve for x. Possible answer: Solve the system by substitution.

Discuss how to convert the answer in **Problem 2** to minutes. $\frac{1}{30}$ h $= \frac{1}{30}$ of 60 min; $\frac{1}{30} \cdot 60 = 2$ min